Quantification in Dynamic Semantics

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Abstract

In dynamic semantics three styles of quantification have been proposed that can be seen to involve two different ways of interpreting free and quantified variables:

- Variables as denoting single partial objects;
- Variables as ranging over a number of alternative total objects.

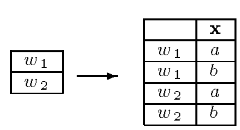
I will show that the first view leads to problems of underspecification and the second to problems of overspecification. I will propose a new style of dynamic quantification in which variables are interpreted in a way which avoids these problems:

- Variables as ranging over a number of alternative definite objects (concepts).

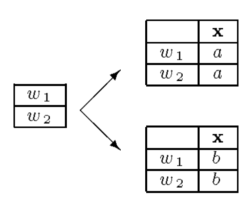
By relativizing quantification to ways of conceptualizing the domain, we avoid the cardinality problems which arise if quantification is over concepts rather than objects.

Background and Motivation

In dynamic semantics, sentences characterize transitions across a space of information states. Information states are generally defined as sets of possibilities (here world-assignment pairs) and meanings describe shifts from states to states: updating with sentences may lead to smaller states in which possibilities have been eliminated, or to richer ones in which new discourse items have been added. Atoms, for example, yield smaller states resulting from eliminating those world-assignment pairs that do not satisfy them. Existential sentences lead to richer states: $\exists x\phi$ adds x and selects a number of possible values for it; the fact that in the output state(s) x is defined means that recurrences of x in later sentences can refer anaphorically. Information about variables is generally modelled in one of the two following ways:
1. Variables are interpreted as single *partial* objects.[1] The introduction of new items is defined in terms of *global extensions* that involve adding fresh variables and assigning them as possible values all elements of the universe.
2. Variables are taken to range over a number of *total* objects. The introduction of a new item is defined in terms of *individual extensions* that lead to the states resulting from adding a variable and assigning it as value a single element of the universe.[2]



1. Global Extension



2. Individual Extension

[0] Special thanks to: David Beaver, Paul Dekker, Jelle Gerbrandy, Jeroen Groenendijk and Andreas Liu.

[1] Partial objects are functions that assign to each possibility in a state the value of the corresponding variable in that possibility. A partial object is total if it is a constant function. For a formal definition cf. [2].

[2] In the pictures the universe consists only of two individuals a and b.

Global extensions yield unique output states, whereas individual extensions produce as many different outputs as there are members of the universe. This involves splitting up the initial state into different alternatives: later sentences will be processed with respect to each of them in a parallel fashion.

In the literature three different interpretations have been proposed for the dynamic existential quantifier that involve one or the other way of interpreting free[3] or quantified variables:

Random Assignment (RA) is the standard interpretation procedure for the dynamic existential quantifier. It is defined in terms of global extension. In this way quantified and free variables are interpreted uniformly as single indefinite partial objects, where further updates will tend to make these objects more definite and less partial. (Cf. [2,7,9,11].)

Slicing (SL) is defined in terms of individual extension: it involves splitting up the update procedure, so that the individuals that a variable can take as possible values are considered one by one, as disjunct alternatives, and not all at once. In this way, quantified and free variables are interpreted uniformly as ranging over a number of alternative total objects, where further updates will tend to eliminate certain alternatives. (Cf. [5].)

Moderate Slicing (MS) follows the slicing procedure as long as we are inside the syntactic scope of a quantifier, but lumps the remaining alternatives together once we are outside its scope. In this way, quantified variables range over a number of alternative total objects, whereas free variables are interpreted as single partial objects. (Cf. [1,3,8].)

These different styles of quantification lead to different results only when combined with non-distributive operators,[4] such as epistemic modals (cf. [2,5,8,12]) or presuppositions (cf. [1,10]). In this paper I will consider only epistemic modals. Modal sentences are interpreted in Veltman's style, as consistency tests. Updating with $\Diamond\phi$ involves checking whether ϕ is consistent with the information encoded in the input state σ. If the test succeeds, i.e. if at least one world-assignment pair in σ survives an update with ϕ, then the resulting state is σ itself; if the test fails, the output state is the empty set, the absurd state.

Although the analysis of combinations of quantifiers and non-distributive operators motivated the use of (moderate) slicing instead of random assignment, I will argue that precisely in such contexts critical problems emerge for all three approaches. The reason is that, of the two ways of interpreting variables that play a role in these approaches, the one that treats variables as single partial objects is too weak and leads to *problems of underspecification*, and the other that views them as a number of alternative total objects is too strong and leads to *problems of overspecification*. These problems occur for both quantified and free variables.

Underspecification and Overspecification

Problem 1: Treating variables in the syntactic scope of an existential quantifier as single partial objects has the unacceptable result that $\exists x\Diamond\phi \models \forall x\Diamond\phi$ (Dekker's problem, cf. [2]).[5] The sentences (1a) $\exists x\Diamond Px$(*Someone might be knocking at the door.*) and (1b) $\exists y\neg\Diamond Py$ (*Someone is certainly not knocking at the door.*) contradict each other if we assume RA. The variables x and y, being introduced via global extension, will denote exactly the same single underspecified object, which either

[3] By a 'free' variable, I mean a variable not occurring inside the syntactic scope of a quantifier. Typically, such 'free' occurrences may still be dynamically bound by a quantifier.

[4] Non-distributive operators are those that take the state holistically and not pointwise with respect to the possibilities in it. So it is not surprising that when we update with a non-distributive sentence, it matters which possibilities are lumped together to form a state and which are kept separate during the procedure.

[5] Heim's fat man problem (cf. [10]), predicting wrong presuppositions, has the same source.

verifies the modal sentence (if at least one member of the universe has the property P in some world) or falsifies it. In RA, quantified variables don't vary enough: the one value that a variable can take cannot be considered separately from the others because all the possible values are lumped together.

Problem 2: If we use slicing, problem 1 does not occur, but the total interpretation of free variables that SL involves leads to the loss of a number of attractive properties guaranteed by RA and MS, for instance the consistency[6] of sentences like (2) $\exists x Px \wedge \forall y \Diamond x = y$ (*Someone is knocking at the door. It might be anyone*). If all variables range over alternative total objects, these sentences become contradictory: it is impossible for one individual to be (possibly) identical to all the others (if $|D| > 1$).

Problem 3: The use of moderate slicing avoids the problems noted above, but runs into several others connected with the notions of support and coherence.[7] For example, consider the sequence (3) $\exists x \Diamond Px \wedge \neg Px$ (*Someone might be knocking at the door. She is not knocking at the door*). Intuitively (3) cannot be coherently asserted, but if we treat variables as denoting single partial objects, we can easily find a state that supports it, so (3) comes out not only consistent, but also coherent. Let σ be a state consisting of two possibilities that supports the information that either individual a or individual b is P, but it is not known which. It is easy to show that such a σ supports (3) given MS (or RA). The first conjunct is supported and leads to a state with four possibilities in which both a and b are assigned as possible values to x for each world. Updating with the second conjunct keeps only those two possibilities that assign to x the individuals that are not P. Note that even if the latter update eliminates possibilities, both possibilities in the initial state survive in the final state. So σ supports the sequence and hence the latter is coherent. Note that slicing, which involves a splitting in the interpretation procedure, avoids this problem: the two initial possibilities do not survive together in any of the output states. This kind of example shows that the notion of support in MS (and RA) is not compositional: we have a state that supports a conjunction, whereas the same state updated with the first conjunct does not support the second one.

Problem 4: As for problems with quantified variables, consider the following discourse uttered in a situation in which the identity of the culprit is unknown:[8] (4) *The culprit did it; so it is not the case that anyone might be innocent. But Alfred might be innocent, Bill might be innocent, ... So anyone might be innocent.* This example shows, among other interesting facts, that we do not always quantify over individuals, but sometimes (e.g., if epistemic modals are involved) over typically non-rigid concepts. So (moderate) slicing, and in general classical quantification, which lets variables range over total objects, is not fully adequate.

	RA	**SL**	**MS**
quantified variables	partial $\leadsto$ problem 1	total $\leadsto$ problem 4	total $\leadsto$ problem 4
free variables	partial $\leadsto$ problem 3	total $\leadsto$ problem 2	partial $\leadsto$ problem 3

Proposal

In order to overcome these problems of over- and underspecification, I propose a new style of dynamic quantification that lies between random assignment and slicing, and which treats free and quantified variables in a uniform way. As in slicing, the interpretation will proceed on different parallel levels so that free and quantified variables range over alternative definite members of some domain and do

[6] A sentence ϕ is consistent iff updating with ϕ does not always result in the empty set.

[7] A state σ supports a sentence ϕ iff each possibility in σ survives in the state resulting from updating σ with ϕ. A sentence is coherent iff there is a non-empty state that supports it. For formal definitions cf. [8].

[8] For more about this kind of example cf. [6].

not denote single indefinite objects; in this way variables will vary enough to avoid the underspecification problems 1 and 3. On the other hand the overspecification problems 2 and 4 are solved by allowing not one but many ways of identifying the objects we quantify over: different domains will arise from different ways of structuring conceptually the universe of discourse. Each conceptualization that covers the whole universe and does not consider any individual more than once will provide a suitable candidate for the domain of quantification. In this way quantifiers may range not only over the set of total objects but also over sets of non-rigid concepts.

Subjects in a State

A first step towards a solution consists in recognizing the difference between the partial objects that caused the underspecification problems, and the still partial but definite objects whose absence from the domain of quantification led to the overspecification problems. The notions defined in this section will provide a way of distinguishing them.

I assume two levels of objects: the individual elements of the universe of discourse that are given once and for all, and the inhabitants of the states. These inhabitants are structured, partial entities introduced as subjects in conversation; they can change, for instance by growing less partial, as the conversation proceeds. So I extend Dekker's definition of partial objects (cf. [2]) and call a *subject* in an information state any mapping from the possibilities (world-assignment pairs) in the state to the individuals in the universe of discourse. Note that besides explicitly introduced discourse items, potential items also count as subjects in a state.

Among the subjects, we can distinguish *rigid subjects* and *(in)definite subjects*. Rigid subjects are the constant functions among the subjects. Intuitively, they represent the elements of the universe of discourse at the level of the state. Definite subjects are those assigning the same value to all possibilities that share the same world. They are contextually restricted (individual) concepts. They are *definite* in that they have a single value relative to a single world, but *partial* in that they may have different values relative to different worlds. Indefinite subjects are subjects that are not definite, i.e. those assigning different values to possibilities with the same factual content.

In random assignment, variables denote possibly indefinite subjects (*partial*). In slicing, variables range over rigid subjects (*total*). I will let variables range over definite subjects (*definite*).

	x
w_1	a
w_1	b
w_2	a
w_2	b

1. partial

	x
w_1	a
w_2	a

	x
w_1	b
w_2	b

2. total

	x
w_1	a
w_2	a

	x
w_1	a
w_2	b

	x
w_1	b
w_2	a

	x
w_1	b
w_2	b

3. definite

However, as is evident from the picture above, the set of all definite subjects in a state cannot serve as the quantificational domain. The fact that there are strictly more concepts in a state than individuals in the universe of discourse would create problems, even in a language as poor as ours (without numerals and the like). For instance, a sentence like *Someone did it, but anyone might be innocent.* would come out inconsistent because if the first conjunct is supported, there will be an element in our quantificational domain that falsifies the universal sentence, namely the concept corresponding to the one who did it.

To avoid these problems, I introduce the notion of a conceptual cover that will provide a suitable way of restricting contextually the set of concepts.

Conceptualizations

Given a model consisting of a non-empty set W of worlds and a non-empty set D of individuals, I will call a *conceptualization* any set of functions from W to D. A conceptualization is a way of structuring the domain. In principle, any conceptualization may constitute a domain of quantification given the right context, but I propose two reasonable conditions that give rise to the desired cardinality results: *exhaustiveness* and *disjointness*. A conceptualization is exhaustive if every individual in D is considered at least once in each world, and it is disjoint if no individual is considered more than once in each world, i.e. if its elements do not overlap. I will call any conceptualization that satisfies both conditions a *conceptual cover*.

Definition 1 [Conceptual Cover] Let $\mathcal{M} = \langle W, D\rangle$. The set $\mathcal{C}_{\mathcal{M}}$ of conceptual covers on $\mathcal{M}$ is defined as:
$\mathcal{C}_{\mathcal{M}} = \{CC \subseteq D^W \mid \forall w \in W : \forall d \in D : \text{ there is a unique } c \in CC : c(w) = d\}$.

By exhaustiveness and disjointness we get the desired cardinality results:
Fact 1. For any conceptual cover $CC \in \mathcal{C}_{\langle W,D\rangle}$, it holds that $|CC| = |D|$.

Among the conceptual covers we find the set of all rigid individual concepts:
Fact 2. $RC = \{c \in D^W \mid \forall w, w' \in W : c(w) = c(w')\}$ is in $\mathcal{C}_{\langle \mathcal{W},\mathcal{D}\rangle}$.
Note, however, that RC is just one among many possible CC.[9]

My proposal is to let variables range over the elements of a contextually-supplied conceptual cover. To do this I need to define an operation that extends information states in the appropriate way.

Definition 2 [Information States] Let $\mathcal{M} = \langle D, W\rangle$ be a model for a language $\mathcal{L}$. Let $\mathcal{V}$ be the set of individual variables in $\mathcal{L}$. The set $\Sigma_{\mathcal{M}}$ of information states based on $\mathcal{M}$ is defined as $\Sigma_{\mathcal{M}} = \bigcup_{X \subseteq \mathcal{V}} \mathcal{P}(W \times D^X)$.

A state is a set of world-assignment pairs in which the assignments share the same domain. C-extensions are operations over states.[10]

Definition 3 [c-extensions] For $c \in D^W$: $\sigma[x/c] = \{i[x/d] \mid c(w_i) = d \;\&\; i \in \sigma\}$.

C-extensions lie between global and individual extensions: they introduce fresh variables and interpret them as certain definite subjects. Dynamic quantifiers are defined in terms of c-extensions; they range over elements of a contextually-given conceptual cover and not (or only indirectly) over the individuals in the universe. In this way, quantification is relativized to ways of conceptualizing the domain. The objects we quantify over (talk or think about) are not given atoms, but are structured, possibly partial entities arising from our ways of organizing conceptually our own experience. Dynamic quantification is defined as follows:[11]

Definition 4 [Quantification] $\sigma[\exists_{CC} x]_a \sigma'$ iff $\sigma' = \sigma[x/c]$ for some $c \in a(CC)$.

CC is a free variable ranging over conceptual covers whose value is supplied by a, which represents the pragmatic context. The fact that each quantifier occurs with its own index allows different quantifiers to range over different domains. Shifts of

[9] Note that given $M = \langle D, W\rangle$ there are $(|D|!)^{|W|-1}$ conceptual covers on M.
[10] The operation $[x/d]$ adds a new variable and assigns it as value the individual d. Cf. [2]).
[11] The universal quantifier is defined in terms of the existential quantifier and negation.

conceptualization are quite exceptional and should be strongly motivated by the context. In (4) above, for example, the shift is needed to preserve consistency and is suggested by the explicit introduction of some of the elements of the new conceptualization.

Slicing and the classical theory of quantification arise as a special case, namely when $CC = RC$, while RA and MS can be defined as derived notions:

$$\begin{array}{rll} \sigma[\exists x\phi]_{RA}\sigma' & \text{iff} & \cup_{c\in RC}\{\sigma[x/c]\}[\phi]\sigma'; \\ \sigma[\exists x\phi]_{SL}\sigma' & \text{iff} & \sigma[x/c][\phi]\sigma' \text{ for some } c \in RC; \\ \sigma[\exists x\phi]_{MS}\sigma' & \text{iff} & \sigma' = \cup_{c\in RC}\{\sigma'' \mid \sigma[x/c][\phi]\sigma''\}. \end{array}$$

I conclude by stating the other semantic clauses and the definition of support.[12]

Definition 5 [The Rest of the Semantics]

$$\begin{array}{rll} \sigma[Rt_1, ..., t_n]_a\sigma' & \text{iff} & \sigma' = \{i \in \sigma \mid < i(t_1), ..., i(t_n) > \in w_i(R)\}; \\ \sigma[\neg\phi]_a\sigma' & \text{iff} & \sigma' = \sigma - \{i \in \sigma \mid \exists\sigma'' : \sigma[\phi]_a\sigma'' \;\&\; i \leq \sigma''\}; \\ \sigma[\Diamond\phi]_a\sigma' & \text{iff} & \sigma' = \{i \in \sigma \mid \exists\sigma'' \neq \emptyset : \sigma[\phi]_a\sigma''\}; \\ \sigma[\phi \wedge \psi]_a\sigma' & \text{iff} & \exists\sigma'' : \; \sigma[\phi]_a\sigma''[\psi]_a\sigma'. \end{array}$$

Definition 6 [Support] $\sigma \models_a \phi$ iff $\exists\sigma' : \; \sigma[\phi]_a\sigma' \;\&\; \forall i \in \sigma : i \leq \sigma'$.

Conclusion

Since only definite subjects may constitute interpretations of variables, the problems of underspecification do not occur. At the same time, since even non-rigid conceptual covers may provide the quantificational domain, the overspecification problems are also avoided. More specifically, we are able to account for examples like (4) above that involve a shift of conceptualization within the same discourse, since different occurrences of quantifiers may range over different domains.

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[12] A possibility i survives in a state σ, $i \leq \sigma$ iff $\exists j \in \sigma$ such that j is the same as i except for the possible introduction of new variables. For a formal definition cf. [2].