Knowing whether A or B

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Abstract. The paper examines the logic and semantics of knowledge attributions of the form "*s* knows whether A or B". We analyze these constructions in an epistemic logic with alternative questions, and propose an account of the context-sensitivity of the corresponding sentences and of their presuppositions.

Keywords: knowledge, alternative questions, contextualism, awareness, relevant alternatives, attention, presupposition

1. Introduction

The aim of this paper is to discuss the semantics of knowledge attributions of the form "*s* knows whether A or B", where the complement "whether A or B" expresses an alternative question, as in "Bob knows whether Mary is French or Italian". Our goal is to provide an account of the logic of these epistemic constructions, and of the context-sensitivity of the corresponding sentences.

It is standard in linguistic theory to distinguish the polar and alternative readings of disjunctive questions, e.g. (von Stechow, 1991). Under the polar reading, a direct question of the form "Is Mary French or Italian?" calls for a yes or no answer. The polar reading can be forced in English by asking "Is Mary either French or Italian?". For the alternative reading, by contrast, the question cannot be answered by yes or no and has to be answered by a sentence like "Mary is French", or "Mary is Italian", namely by providing information about the truth and falsity of the respective disjuncts.

There is still some debate in the literature about the answerhood conditions of alternative questions, and in consequence, about the conditions under which a subject can be said to know whether A or B. In a recent paper (Schaffer, 2007), J. Schaffer has argued that in a context in which a subject *s* sees someone on TV, who is actually George Bush, but such that *s* is not able to discriminate between Bush and Will Ferrell (because Ferrell is a very good Bush impersonator), and yet is able to see that it is not Janet Jackson, (1-a) below should be judged false, but (1-b) should count as true:

(1) a. *s* knows whether George Bush or Will Ferrell is on TV
b. *s* knows whether George Bush or Janet Jackson is on TV.

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The intuition reason for the truth of (1-b), according to Schaffer, is that the question "Is Bush or Janet Jackson on TV?" is easier for s to answer than the question "Is Bush or Will Ferrell on TV?". In our view, however, although (1-a) should be incontrovertibly false in the scenario, ordinary intuitions are about (1-b) are less stable. In our opinion, all that s really knows is that *Janet Jackson is not on* TV , which may not be sufficient to answer the question "Is Bush or Jackson on TV?".

More formally, assuming the partition theory of questions of Groenendijk and Stokhof (Groenendijk and Stokhof, 1984), an answer of the form "Janet Jackson is not on TV" counts only as a *partial answer* to the question "Is Bush or Janet Jackson on TV?". For s to know the complete answer he should know more, namely also that Bush is on TV. The partial answer "Janet Jackson is not on TV" would count as complete if one presupposed that exactly one of the two disjuncts had to be true. In principle, however, there is no more reason to think that "s knows whether Bush or Janet Jackson is on TV" is true than there is to think that "s knows whether Ferrell or Janet Jackson is on TV" is true. In other words, s's ignorance about who exactly *is* on TV seems to override s's partial knowledge about who is *not* on TV.

Despite this, we agree with Schaffer that there is a sense in which, if s is allowed to *ignore* the possibility that Ferrell might be on TV, then s can be said to know whether Bush or Janet Jackson is. In a previous paper (Aloni and Égré, 2008) we discussed several ways of making sense of Schaffer's contextualist idea, and offered to clarify the meaning of knowing-whether constructions. Two issues were left for further investigation. The first concerns the grounds on which an agent is allowed to ignore possibilities (from an internal perspective, and from the perspective of an external ascriber). In this paper we propose to relate the status of "knowing whether" sentences to *attention to possibilities*, to clarify in which cases an agent who is not attending to a possibility can still safely ignore that possibility, and be ascribed knowledge (Lewis, 1996). A second, more linguistic issue that we only partially investigated concerns the nature of the *presuppositions* that go with the use of alternative questions under the verb *know*.

We examine both these issues in turn in the paper. Before doing that, we lay out the main elements we need in order to analyze "knowing whether" constructions in epistemic logic.

2. Knowing whether

Logicians are familiar with the usual paraphrase in epistemic logic of expressions of the form "Bob knows whether (or not) Mary is French" in terms of "knowing that" and disjunction, namely as "Bob knows that Mary is French or Bob knows that Mary is not French":

(2) a. Bob knows whether (or not) Mary is French.
b. $Kp \vee K\neg p$

When considering an expression of the form "Bob knows whether Mary is French or Italian", however, the situation is more complicated than it may seem at first. The reason is that such a sentence is typically ambiguous. One way of paraphrasing the sentence, by analogy with the previous case, is as meaning: "Bob knows that Mary is French or Italian, or Bob knows that it is not the case that Mary is French or Italian".

(3) a. Bob knows whether (or not) Mary is French or Italian.
b. $K(p \vee q) \vee K\neg(p \vee q)$

This paraphrase corresponds to what is known as the *polar reading* of embedded disjunctive questions. Bob knows whether Mary is French or Italian in that sense if Bob can answer by "yes" or "no" to the direct question "Is Mary French or Italian?". If pressed to answer what Mary's nationality is, however, Bob may only be able to respond "French or Italian", without knowing *which*.

A second, distinct reading for these disjunctive questions is what is known as the *alternative reading*. The expected answer for the direct disjunctive question in this case is not "Yes" or "No", but will typically be "She is French" or "She is Italian". In the embedded case, "whether" can no longer replaced by "whether or not" to express alternative questions, as "or not" forces the polar reading (Larson, 1985).

Surprisingly, analyses diverge on what the paraphrase of "Bob knows whether Mary is French or Italian" ought to be in the alternative reading. One obvious candidate is the disjunction "Bob knows that Mary is French or Bob knows that Mary is Italian":

(4) a. Bob knows whether Mary is French or Italian
b. $Kp \vee Kq$

This analysis requires that Bob be able to respond to the question by giving at least one of the answers "French" or "Italian". This account seems inadequate, however. For instance, suppose that Mary is neither French nor Italian, but German, and Bob knows it. Then what Bob knows can be represented by the formula $K(\neg p \wedge \neg q)$. In that case, (4-b) will be false. Intuitively, however, we would not want to say that (4-a) is false, given that he can give the correct and maximally informative answer "neither".

A dual problem arises if Mary happens to be both French and Italian. Let us assume that Bob only knows Mary to be French, but fails to realize she is also Italian. He would answer "French" to "Is Mary French or Italian?". In such a situation, (4-b) will indeed be true, but it is not obvious that (4-a) should count as true. Although Bob's answer is not wrong in that case, it would still be *incomplete* with respect to the situation under discussion, and possibly incorrect in other cases. Indeed, imagine that Bob knows Mary is French but wrongly believes that she is not Italian: (4-b) will still count as true, but then we would definitely be reluctant to say that Bob knows whether Mary is French or Italian if his complete answer was "She is French and not Italian".

Our favored analysis of alternative questions under *know* follows the lead of the partitional analysis of questions proposed more generally by (Groenendijk and Stokhof, 1984). To know whether Mary is French or Italian, in their framework, means to be able to respond with one of the answers "French", "Italian", "both" or "neither", whichever of these options is the true one. Thus, our desired paraphrase for embedded alternative questions is:

(5) a. Bob knows whether Mary is French or Italian.
b. $Kp\neg q \vee K\neg pq \vee Kpq \vee K\neg p\neg q$

One of the features of this analysis of embedded alternative questions is that it is truth-conditionally equivalent to $(Kp \vee K\neg p) \wedge (Kq \vee K\neg q)$ (assuming the standard Kripke semantics for the K operator, see Section 2). In other words, the analysis predicts that knowing whether p or q is truth-conditionally equivalent to knowing whether p and knowing whether q. Thus, Bob knows whether Mary is French or Italian if and only if Bob can correctly answer each of the polar questions "Is Mary French?" and "Is Mary Italian?". As the reader can check, this analysis avoids both of the problems we raised for the analysis in (4-b).

Before considering further examples, we should mention one other candidate analysis for "knowing whether p or q" under the alternative reading. A common interpretation of "Bob knows whether Mary is French or Italian" is "Bob knows which of the two nationalities 'French' and 'Italian' Mary has, *assuming she has only one of them*." This can be expressed as:

(6) a. Bob knows whether Mary is French or Italian.
b. $Kp\neg q \vee Kq\neg p$.

As the reader can check, this analysis can be seen as a strengthening of (4-b), in which the two alternatives p and q are taken to be mutually exclusive. It is also a strengthening of the partitional analysis in

(6-b). Unlike (4-b), but like (5-b), this analysis entails that if you know whether p or q, you know whether p and you know whether q. But again the objections we raised against (4-b) can be raised in principle against (6-b). If Bob knows that Mary is both Italian and French, then (6-b) is simply false, but logically speaking we would not want to say that Bob does not know whether Mary is Italian or French in that case.

The intuitive appeal behind the strengthened truth-conditions in (6-b) may be due to the fact that, in ordinary language, we take an alternative question to implicate or to presuppose that exactly one of the alternatives is true. Thus, someone who asks "Is Mary French or Italian?" generally assumes that Mary has at least one and at most one of the two nationalities. On our account, this fact relates to the use and pragmatics of alternative questions, and should not directly be part of their semantics, nor of the truth conditions of "knowing whether" constructions. We say more about the status of these assumptions Sections 4 and 6 below.

Fundamentally, therefore, our working assumption in this paper is that (5-b) captures exactly the truth-conditions of "knowing whether" constructions with alternative questions. In the next section, we introduce a combined logic of knowledge and questions in which the partitional analysis of "knowing whether A or B" constructions can be formally derived.

3. An Epistemic Logic with Alternative Questions

The logic introduced in this section is a version of propositional epistemic logic with questions. A question is a sentence of the form $?\vec{p}\,\phi$ where $\vec{p}$ is a sequence $p_1, \ldots, p_n$ of propositional variables. In the language we distinguish between propositional variables p and propositional constants a.

Definition 1. (Syntax)

$$\phi =: a \mid p \mid \neg\phi \mid (\phi \wedge \psi) \mid K\phi \mid ?\vec{p}\,\phi \mid \phi = \psi$$

A model M is a quadruple (W, R, P, V), where W is a non-empty set of possible worlds, R is an equivalence relation on W, P is a non-empty set of subsets of W (i.e. of propositions) satisfying a number of properties (Fine, 1970), and V is a valuation function which associates an element α of P to each propositional constant a in the language. An assignment function g associates an element of P to each propositional variable p.

The semantics, using double-indexing to capture question meanings (Lewis, 1982; Groenendijk and Stokhof, 1982), is spelled out in terms of a satisfaction relation $\models_g$, which may hold between a model M and a pair of worlds w, v, on the one hand, and a formula ϕ, on the other. The second index plays no role in the first five clauses. It is used to keep track of the actual world when evaluating a question under a knowledge operator.

Definition 2. (Semantics)

$$\begin{array}{rcl}
M,w,v \models_g a & \text{iff} & w \in V(a) \\
M,w,v \models_g p & \text{iff} & w \in g(p) \\
M,w,v \models_g \neg\phi & \text{iff} & \text{not } M,w,v \models_g \phi \\
M,w,v \models_g \phi \wedge \psi & \text{iff} & M,w,v \models_g \phi \ \& \ M,w,v \models_g \psi \\
M,w,v \models_g \phi = \psi & \text{iff} & \forall w' : M,w',v \models_g \phi \text{ iff } M,w',v \models_g \psi \\
M,w,v \models_g ?\vec{p}\,\phi & \text{iff} & \forall \vec{\alpha} \in P^n : M,w,v \models_{g[\vec{p}/\vec{\alpha}]} \phi \text{ iff } M,v,v \models_{g[\vec{p}/\vec{\alpha}]} \phi \\
M,w,v \models_g K\phi & \text{iff} & \forall w' : wRw' \ \Rightarrow \ M,w',w \models_g \phi
\end{array}$$

Disjunction $\vee$, implication $\rightarrow$, are defined as standard in terms of $\neg$ and $\wedge$. Truth and entailment are defined as follows:

Definition 3. (Truth and entailment)

(i) $M,w,v \models \phi$ iff $\forall g : M,w,v \models_g \phi$;

(ii) $\phi_1,\ldots,\phi_n \models \psi$ iff for all M,w,v :
$$M,w,v \models \phi_1,\ldots,M,w,v \models \phi_n \Rightarrow M,w,v \models \psi.$$

The double-indexing plays a role only in the clause for questions (and knowledge-*wh*). If ϕ does not contain any question, we can easily prove that for all $w,v \in W$: $M,w,v \models_g \phi$ iff $M,w,w \models_g \phi$.

Alternative and polar disjunctive questions are represented as:[1]

(7) a. Alternative questions: $?p(p \wedge (p = a \vee p = b))$
b. Polar disjunctive questions: $?(a \vee b)$

The alternative question representation can be paraphrased as "Which of either proposition is true: a or b?" (Larson, 1985). This paraphrase suggests that knowing whether a or b will prove equivalent to $Ka \vee Kb$, but our logic derives the stronger, partitional meaning. Crucial for this result is the double indexing technique employed here. In what follows we will use $?(a \vee_{\mathrm{A}} b)$ as short for $?p(p \wedge (p = a \vee p = b))$.

The following equivalences hold in this logic (we prove one direction of (8-c) in the appendix, as an illustration of the system):

[1] Constituent questions like 'Who called?' can be represented in a predicate version of the present system by sentences like $?p(p \wedge \exists x\ Cx = p)$.

(8) a. $K\ ?a \equiv Ka \vee K\neg a$
b. $K\ ?(a \vee b) \equiv K(a \vee b) \vee K\neg(a \vee b)$
c. $K\ ?(a \vee_{\mathrm{A}} b) \equiv K?a \wedge K?b$
d. $K?a \wedge K?b \equiv K(a \wedge \neg b) \vee K(\neg a \wedge b) \vee K(a \wedge b) \vee K\neg(a \vee b)$

4. Schaffer's puzzle

In the previous section we examined different ways of analyzing epistemic constructions with "whether"-complements. We now turn to the discussion of Schaffer's puzzle and the problem of context-sensitivity of *knowing whether* sentences. Schaffer's scenario is one in which a subject, Bob, sees a particular person on TV, who happens to be George Bush. By assumption, Bob is not able to discriminate between George Bush and a clever impersonator like Ferrell, but can discriminate between George Bush and Janet Jackson. In particular, Bob can see that the person on TV is not Janet Jackson. Schaffer's puzzle concerns the semantic judgments we should issue about the pair of sentences (1-a) and (1-b), here repeated as (9-a) and (9-b):

(9) a. Bob knows whether George Bush or Will Ferrell is on TV
b. Bob knows whether Bush or Janet Jackson is on TV.

Schaffer's original intuition about this example is that while (9-a) should be false, (9-b) should be true. That (9-a) should be false is intuitive: since s cannot discriminate between Bush and Ferrell, s does not know which of Bush and Ferrell is on TV. Arguably, Schaffer's intuition for the truth of (9-b) can in principle be established similarly: since s can discriminate in principle between Bush and Janet Jackson, it should follows that s knows which of Bush and Janet Jackson is on TV.

In (Aloni and Égré, 2008), we argued that the situation is probably more complicated as regards (9-b). Something we noticed is that the sentence can equally be judged false or infelicitous by competent and rational speakers. Nevertheless, we admit that it can also be judged true. How can this be? Here we present the different options in turn.

The "false" judgment. First, the partition semantics for alternative questions that we assumed in the previous section makes the basic prediction that (9-b) should in fact be *false* in the scenario under discussion. Indeed, as explained earlier, $K?(b \vee_{\mathrm{A}} j)$ is true if and only if $K?b \wedge K?j$ is also true. But if indeed Bob cannot discriminate Bush from Ferrell, then this should entail $\neg K?b$, namely that Bob does not know whether Bush is on TV or not, and therefore $\neg K?(b \vee_{\mathrm{A}} j)$. Another way to put it is to say that all Bob really knows, by assumption, is that the person on TV is not Janet Jackson, namely $K\neg j$. But this is not enough for Bob to know positively *who* is on TV. For $K?(b \vee_{\mathrm{A}} j)$ to be true in this case, the stronger proposition $K(b \wedge \neg j)$ must be true, namely Bob must also know *that Bush is on* TV . But this is not the case, given Bob's inability to ascertain that it is not Ferrell.

The "undefined" judgment. The judgment that (9-b) is false does not necessarily match what logically competent speakers of English report about the situation. In fact, one can observe a contrast between:

(10) a. It is not true that Bob knows whether Bush or Janet Jackson is on TV.
b. Bob does not know whether Bush or Jackson is on TV.

Some of the subjects we asked (including ourselves) feel more readily inclined to judge (10-a) true than (10-b). In the case of (10-b), what we feel is that the sentence is just as inappropriate as its unnegated counterpart (9-b). How can this be? One intuitive reason we see for the infelicity judgment is the following: it is odd to utter (10-b) if Bob knows that Janet Jackson is not on TV. Intuitively, (10-b) suggests that Bob should be equally uncertain about Bush and about Janet Jackson. In (Aloni and Égré, 2008) we observed that this fact can be derived if one adds to the basic truth-conditions of *knowing-whether* sentences a presupposition of symmetry with regard to the alternatives present in an alternative question. In the case of two alternatives, the principle can be expressed as follows:

(11) a. Bob knows whether A or B presupposes that Bob knows whether A iff Bob knows whether B.
b. $K?(a \vee_{\mathrm{A}} b)$ is true or false in w, v provided $(K?a \leftrightarrow K?b)$ is true in w, v; it is undefined otherwise.

It is immediate that (9-b) will be undefined, since by assumption $K?j$ is true, and $K?b$ false in the scenario under discussion. One important consequence of the principle is that it allows us to derive the following fact:

(12) $\neg K?(a \vee_{\mathrm{A}} b) \models \neg K?a \wedge \neg K?b$

This means that in every model in which $\neg K?(a \vee_{\mathrm{A}} b)$ is (defined and) true, given the symmetry presupposition, $\neg K?a \wedge \neg K?b$ is also true. And indeed, from "Bob does not know whether Bush or Janet Jackson is on TV", one readily infers that "Bob does not know whether Bush is on TV, *and* Bob does not know whether Janet Jackson is on TV". Without the symmetry presupposition only the following weaker entailment holds in our logic:

(13) $\neg K?(a \vee_{\mathrm{A}} b) \models \neg K?a \vee \neg K?b$

The symmetry presupposition is a particular case of a more general principle that Chemla calls the principle of epistemic similarity for disjunctive sentences (Chemla, 2008). We shall not try to motivate this further here, but discuss it in greater detail in Section 6. The important fact to bear in mind, from a logical point of view, is that this presupposition is probably the weakest we can add to the partitional semantics in order to support the intuition that (9-b) ought to be undefined.

The "true" judgment. Besides judgments of undefinedness and falsity for (9-b), there is a point to the intuition that the sentence can be true, as Schaffer submits. We gave an intuitive reason above already: because Bob is able in principle to discriminate between Bush and Janet Jackson, his knowledge that it is not Jackson may be sufficient relative to those two alternatives to conclude that the person on TV is Bush. In (Aloni and Égré, 2008), we described this using a mechanism of *topical restriction*. Bob's knowledge which of the alternatives holds is evaluated in his epistemic state restricted to the propositions mentioned in the question. A static approximation of the context-sensitive truth-conditions stated in (Aloni and Égré, 2008) is the following:

(14) $M, w, v \models_g K?(a \vee_{\mathrm{A}} b)$ iff $\forall w' : wRw'$ and $(M, w', v \models a$ or $M, w', v \models b)$, $M, w', v \models_g ?(a \vee_{\mathrm{A}} b)$

This semantics predicts that "Bob knows whether the person on TV is Bush or Jackson" is true then, even though Bob cannot discriminate in principle between Bush and Ferrell. And it preserves the fact that "Bob knows whether the person on TV is Bush or Ferrell" is false, since the restriction then is idle.

The restriction mechanism in (14) raises two issues. The first is linguistic and concerns the pragmatic or semantic character of this restriction. If such a restriction takes place, does it take place systematically? If it were so, one should be able to say in the same context:

(15) a. Bob knows whether the person on TV is Bush or Jackson, although he does not know whether the person on TV is Bush or Ferrell.
b. $K?(b \vee_{\mathrm{A}} j) \wedge \neg K?(b \vee_{\mathrm{A}} f)$.

But this should imply that "Bob knows whether A or B" is synonymous with "*assuming A or B is true*, Bob knows which of the two is true". This is not obviously the case, however, and in one and the same context (15-a) sounds like a near contradiction. This suggests that the mechanism in question is fundamentally pragmatic, and that more contextual-dependence is at play (Aloni and Égré, 2008).

The second issue is epistemological. Suppose Bob pays attention to all the alternatives present, namely Bush, Jackson and Ferrell. Then Bob's uncertainty regarding the Bush-Ferrell pair might affect his confidence that the person on TV is Bush when asked "Is it Bush or Jackson?". But are the judgments the same if Bob is not even aware of the existence of Ferrell, but only of that of Jackson and Bush? Then (15) may more easily be judged true. This suggests that more should also be said about the attention the agent pays to the alternatives. In the following sections our aim will thus be to characterize more tightly the contextual parameters that influence our judgments of truth value for a sentence like (9-b).

5. Attention and Relevance

As is well known from the epistemological literature, one should distinguish precisely subject's factors and attributor's factors in discussing the semantics of knowledge sentences (Stanley, 2005). In this section we examine Schaffer's scenario in the light of two such factors that we call *attention* and *relevance*. The surprising conclusion of this section will be that a significant part of Schaffer's puzzle need not have anything to do with the particular properties of knowing-*wh* constructions after all. A model of attention and relevance, motivated primarily by the semantics of knowing-*that* sentences, is enough to give us both the 'true' and 'false' judgements we considered in the previous section. We set aside the undefinedness judgments until the end of this section; that is, we make no use of the symmetry principle (11) and keep the logic entirely bivalent for the purpose of teasing apart the 'true' and 'false' judgments first.

5.1. Contextualism and proper ignorance

Attention is a subject-based parameter, which concerns whether or not a subject is consciously attending to a particular epistemic possibility. For instance, Bob may fail to attend to the possibility that the person on TV is Ferrell, either because he does not think about Ferrell, or because he has never heard of Ferrell. Relevance on the other hand is a normative, ascriber-based parameter, which concerns whether it is permissible or not, from an external ascriber's standpoint, to ignore a particular possibility.

1970As contextualists such as Dretske and Lewis have argued, in some cases some epistemic possibilities can be irrelevant and may be "properly ignored" (for instance very remote far-fetched sceptical possibilities). Cases in which a subject fails to attend to *relevant* possibilities undercut his knowledge, but conversely, when these possibilities may be properly ignored, the subject's inattention to them will leave his knowledge unscathed. To account for the force of particular instances of the sceptic's argument, if the subject does not in fact ignore a possibility (for instance when it is mentioned as part of a sceptical argument) he is thereby no longer properly ignoring it and it may likewise undercut his knowledge.

Based on this idea, the point of this section is to show that Schaffer's intuition, according to which the sentences $\neg K?(b \vee_{\mathrm{A}} f)$ and $K?(b \vee_{\mathrm{A}} j)$ can be true together, should be examined in relation to these two parameters; in particular, relative to whether the possibility that Ferrell is on TV can be properly ignored or not, and relative to whether it is attended to by Bob or not.

According to the partitional semantics of Section 2, for Schaffer's intuition to hold we must have simultaneously $\neg K?f$, $K?b$ and $K?j$. The prediction we shall derive is that this is only possible (under our running assumptions) when the Ferrell possibility is both irrelevant and not attended to. In all other cases, the relevance of f, or the fact that it is attended to by Bob, forces both $K?b$ and $K?f$ to be false together.

5.2. Modelling attention and relevance

In order to model attention and relevance, we need to enrich the models introduced so far. Relevance in our approach is modelled by associating to each world a set of relevant alternatives, possibilities that cannot be properly ignored at the world. Attention, on the other hand, is defined by two components: by the possibilities the agent is actually entertaining, and also by the sentences the agent consciously considers. Attention to sentences is needed to avoid the classic problem of logical omniscience, or of epistemic closure. For instance, Bob may attend to the 'metaphysical truth' that Bush is not Janet Jackson without attending to the fact that Bush is not Ferrell, even though both are true in exactly the same worlds (namely all of them).[2]

We define an *attention and relevance* model (for a single agent) as a structure $(W, R, S, E, \mathcal{A}, P, V)$, where (W, R, P, V) is as in Section 2. $S \colon W \to \wp(W)$ associates with each world w the set $S(w)$ of worlds the agent *should* entertain in w (relevant alternatives; possibilities the agent

[2] The model of attention we use is based on a logic of awareness (Fagin and Halpern, 1988); it departs from the standard treatment in that the subject may hold implicit beliefs (or *assumptions* in the sense of (Franke and de Jager, 2008)) about sentences he does not attend to.

may not properly ignore); for each w we require $w \in S(w)$ (without this constraint contextualist knowledge is no longer factive). $E\colon W \to \mathcal{P}(W)$ associates with w the set $E(w)$ of possibilities the agent actually *entertains*; finally $\mathcal{A}$ is a function which to each world w associates a set of propositional constants $\mathcal{A}(w)$, which our agent consciously *attends to*. Let $\mathcal{L}(\mathcal{A}(w))$ be the set of sentences generated from $\mathcal{A}(w)$; we say that the agent attends to ϕ at w if $\phi \in \mathcal{L}(\mathcal{A}(w))$.

Our definition of knowledge, relative to this model, is the following:

$$\begin{array}{lll} M, w, v \models_g K\phi & \text{iff} & \phi \in \mathcal{L}(\mathcal{A}(w)) \text{ and} \\ & & \forall w' \in E(w) \cup S(w)\colon wRw' \Rightarrow M, w', w \models_g \phi \end{array}$$

The definition says that an agent knows that ϕ if firstly he attends to ϕ (we describe *conscious*, rather than *potential* knowledge), and then if ϕ holds in every epistemic alternative that either *is* entertained by the agent or *should* be entertained, by the normative criterion of relevance.[3]

5.3. Predictions

We have to consider four scenarios in which each time the person on TV is Bush. Each time, Bob cannot discriminate in principle between the actual world w_b and the world w_f in which Ferrell is on TV, but is able to discriminate w_j from the other two (the relation R is supposed reflexive, symmetric and transitive). Our scenarios vary along two axes. Along the attention axis, Bob may attend to f or not; if he does, he entertains w_f, otherwise he does not (this is a natural assumption in this scenario; in fact it follows in general from the constraints alluded to in footnote 3). Along the relevance axis, the possibility w_f may either be properly ignored or not.

To get the right intuitions about the relevance axis, imagine two different contexts in which Bob might believe he sees Bush on television. In the first, he is watching a live broadcast from the White House, with Bush addressing the American people. In this setting the idea that Bush is being impersonated by Will Ferrell seems a mere sceptical possibility, along the same lines as the possibility that he is hallucinating the broadcast. We say therefore that $S_1(w_b) = \{w_b\}$ (there is no normative requirement that Bob attends to any non-Bush possibilities).

[3] This semantics will over-generate for complex nested operator constructions such as $K?K?p$. A partial solution is to add the attention and relevant alternative sets as free parameters rather than world-based functions. In either case, however, a number of constraints on possible combinations of these parameters with R and with each other are required. For reasons of space we omit the details.

In a different scenario, he sees footage of Bush making fun of his administration and his own policies. The fact that this behavior is completely outrageous for an American president but perfectly appropriate for a comic impersonator means that Bob would be naive, at best, to discount the possibility that he is seeing Ferrell. We represent this with $S_2(w_b) = \{w_b, w_f\}$ (he is normatively required to entertain the possibility w_f).

Cutting across these two scenarios are the two states of attention Bob may be in. We'll assume he attends to b and j, and entertains w_b and w_j, in both states. But in one he fails to attend to f and ignores w_f: $\mathcal{A}_1(w_b) = \{b, j\}$ and $E_1(w_b) = \{w_b, w_j\}$. In the other he consciously considers the possibility that it is Ferrell: $\mathcal{A}_2(w_b) = \{b, j, f\}$ and $E_2(w_b) = \{w_b, w_j, w_f\}$. All scenarios are summarized in the following table, in which the world of evaluation is w_b:

	$S_1 = \{w_b\}$	$S_2 = \{w_b, w_f\}$
$E_1 = \{w_b, w_j\}$ $\mathcal{A}_1 = \{b, j\}$	$K?(b \vee_{\mathrm{A}} j)$, $K?b$ $\neg K?(b \vee_{\mathrm{A}} f)$, $\neg K?f$	$\neg K?(b \vee_{\mathrm{A}} j)$, $\neg K?b$ $\neg K?(b \vee_{\mathrm{A}} f)$, $\neg K?f$
$E_2 = \{w_b, w_j, w_f\}$ $\mathcal{A}_2 = \{b, j, f\}$	$\neg K?(b \vee_{\mathrm{A}} j)$, $\neg K?b$ $\neg K?(b \vee_{\mathrm{A}} f)$, $\neg K?f$	$\neg K?(b \vee_{\mathrm{A}} j)$, $\neg K?b$ $\neg K?(b \vee_{\mathrm{A}} f)$, $\neg K?f$

Of the four combinations we get, only one supports Schaffer's judgement: S_1 with $\mathcal{A}_1$ and E_1. In this combination Bob ignores w_f, and according to S_1 he is allowed to ignore it. This supports the judgement $K?b \wedge K?j$; since he doesn't attend to f, on the other hand, $K?f$ is false.

In the other three combinations Bob fails to know whether Bush is on TV, but interestingly enough, for different reasons. Whenever he attends to f he is consciously *uncertain* whether Ferrell or Bush is on TV, however when $\mathcal{A}_1$ and E_1 combine with S_2 he *believes* he knows that b, but the normative judgement supplied by S_2 prevents us from agreeing with his belief: he ignores a possibility that he ought to attend to, and which would undercut his knowledge.

More generally, Bob may fail to know at w that some formula ϕ holds for three quite different reasons:

1. Because he is consciously uncertain about ϕ (the most standard reason to fail to know); this underpins $\neg Kb$ whenever $w_f \in E(w)$.

2. Because he fails to consider some non-ϕ possibility that he ought to consider (he believes he knows ϕ but he is wrong). Our scenario combining S_2 with E_1 and $\mathcal{A}_1$ supports $\neg Kb$ for this reason.

3. Because he does not attend to ϕ (he cannot even wonder *whether* ϕ). This is the reason $K(b \to \neg f)$ does not hold for any scenario with $\mathcal{A}_1$, in which he does not attend to f.

5.4. Summary

The strategy of explanation we followed in this section differs significantly from the one we sketched with (14) earlier. The topical restriction strategy outlined in (14) predicts that $K?(b \vee_{\mathrm{A}} j) \wedge \neg K?(b \vee_{\mathrm{A}} f)$ can be true without modification of the semantics for K. But we saw that it makes predictions that are intuitively inconsistent. In particular, $K?(b \vee_{\mathrm{A}} j) \wedge \neg K?(b \vee_{\mathrm{A}} f)$ should then be compatible with both $\neg K?b$ and $\neg K?f$, unless topical restriction should carry over to *knowing whether* sentences involving polar questions.[4]

In contrast, the account put forward in the present section rests on a more fine-grained semantics for knowledge, but makes no modification of the semantics of *knowing whether* clauses proper. The strength of the present account is that Schaffer's conjunction $K?(b \vee_{\mathrm{A}} j) \wedge \neg K?(b \vee_{\mathrm{A}} f)$ is predicted to be true only when f is a possibility that is both ignored and irrelevant, and otherwise false. Furthermore, it derives judgments of truth and falsity about Kb and Kf in a principled way. On this account, the truth of $K?(b \vee_{\mathrm{A}} j) \wedge \neg K?(b \vee_{\mathrm{A}} f)$ is always incompatible with that of $\neg K?b$ and $\neg K?f$.

One important aspect is that the semantics of attention and relevance can accommodate the principle of symmetry we mentioned in the previous section if we want to make sense of the 'undefined' judgments. We would predict that $K?(b \vee_{\mathrm{A}} j)$ would be true in state S_1, E_1, $\mathcal{A}_1$, but undefined everywhere else, as $\neg K?b$ and $K?j$ would hold in all other states. What this means is that the undefined judgments are essentially parasitic on stronger judgments of falsity anyway. In the next section, we say more about the motivations for this principle and competing pragmatic explanations for Schaffer's intuition.

6. Pragmatic inference

In this section we discuss two presuppositions that have been proposed for alternative questions in the literature. Suppose that John knows whether $A_1, \ldots, A_n$. The 'exactly one presupposition' (e.g. (Karttunen,

[4] A dynamic mechanism of topical restriction is presented in (Aloni and Égré, 2008), making room for this possibility. We do not discuss it here, as our focus is on static aspects of the context-sensitivity of knowing whether sentences.

1977)) is that exactly one of the alternatives A_1,..., A_n is true. The 'symmetry presupposition' (Aloni and Égré, 2008) is that John is equally competent about all of the alternatives (he knows whether A_1 iff he knows whether A_2 iff ... and so on).

We will argue that the inference patterns supporting these presuppositions are of a pragmatic character—both can be cancelled under certain circumstances. Our semantic analysis of alternative questions then should not be made dependent on them, otherwise we would fail to explain our intuitions in these cases. Nevertheless, the effects of these pragmatic inferences on natural judgements are real. To explain ordinary uses of alternative questions in discourse, we define a notion of 'pragmatic' entailment which models a pragmatic mechanism of presupposition accommodation.

A presupposition is an inference that holds for both a sentence and its negation, and that, if not satisfied, leads to an infelicity judgement. The oddity of "The King of France is not bald" (a classic example) can be explained as a case of presupposition failure.

In ordinary conversation, however, presupposition failure can be (and often is) repaired by accommodation (Lewis, 1979). One way to model accommodation in our logic is with a pragmatics-sensitive notion of entailment. This notion is sometimes called Strawson entailment (Fintel, 1999), and it is defined by restricting attention to models that satisfy the presuppositions of all the sentences involved.

Definition 4. (Strawson entailment) $\phi_1, \ldots, \phi_n \models_S \psi$ iff $\forall M, w, v$ such that M satisfies the presuppositions of $\phi_1, \ldots, \phi_n$ and ψ: $M, w, v \models \phi_1, \ldots, M, w, v \models \phi_n \Rightarrow M, w, v \models \psi$.

Let us put this definition to work on the two presuppositions.

6.1. Symmetry

Assuming S knows Napoleon was not born in 1869, it may appear altogether infelicitous to say:

(16) S does not know whether Napoleon was born in 1769 or in 1869.

More generally, "S does not know whether A or B" seems to imply "S does not know whether A and S does not know whether B". However, in the situation under discussion, S knows that Napoleon was not born in 1869.

To account for this intuition, (Aloni and Égré, 2008) proposed the following presupposition:

(17) **Symmetry presupposition**

a. John knows whether A_1 or ... or A_n *presupposes*
b. John knows whether $A_1 \Leftrightarrow \ldots \Leftrightarrow$ John knows whether A_n

As explained in Section 4, symmetry failure can also account for the "undefined" judgement in Schaffer's scenario. Sentence (18) is naturally judged infelicitous because Bob has different epistemic attitudes towards the two alternatives mentioned (he knows *that*, so also whether, Jackson is not on TV, but he fails to know whether Bush is on TV).

(18) Bob knows whether Bush or Jackson is on TV.

But when presuppositions fail people normally accommodate, and this is precisely what the notion of Strawson entailment is meant to represent. It is easy to see that if we assume that $K?(b \vee_{\mathrm{A}} j)$ presupposes $K?b \leftrightarrow K?j$, then we can prove the following Strawson entailment:

(19) $K\neg j \models_{\mathrm{S}} K?(b \vee_{\mathrm{A}} j)$

because the standard entailment (20) holds:

(20) $K\neg j, K?b \leftrightarrow K?j \models K?(b \vee_{\mathrm{A}} j)$

It seems then that by adding a mechanism of presupposition accommodation we also have an explanation of the "true" judgement discussed in Section 4. Knowing that Jackson is not on TV pragmatically entails knowing whether Jackson or Bush is on TV.

There is something missing though in this explanation. From "Bob knows that Jackson is not on TV" and "Bob knows whether Jackson or Bush is on TV", we normally conclude that "Bob knows *that* Bush is on TV". Our pragmatic analysis so far, instead, can only derive that "Bob knows *whether* Bush is on TV". This fact shows that there is another pragmatic effect of the use of alternative questions to which we turn in the following subsection.

6.2. Exactly one

Many authors have observed that the use of an alternative question suggests that one and at most one of the alternatives is true. If you know that Mary is German, it seems infelicitous to say:

(21) a. Bob knows whether Mary is French or Italian.
b. Bob doesn't know whether Mary is French or Italian.

The same infelicity appears if Mary happens to be both French and Italian.

These judgements can be explained in terms of a failure of an *exactly one* presupposition:

(22) **Exactly one presupposition**

a. Bob knows whether $A_1, \ldots, A_n$ *presupposes*

b. (i) *At most* one of the $A_1, \ldots, A_n$ is true

(ii) *At least* one of the $A_1, \ldots, A_n$ is true

It is easy to see that if we assume both the symmetry and *exactly one* presuppositions, we have a full explanation of the "true" judgement in Schaffer's scenario. If we are ready to accommodate the presupposition of "Bob knows whether Bush or Jackson is on TV", then from "Bob knows that Jackson is not on TV" we can conclude that "Bob knows that Bush is on TV". The following pragmatic inferences hold:

(23) a. $K\neg j \models_{\mathrm{S}} K?(j \vee_{\mathrm{A}} b)$ (by symmetry)

b. $K\neg j, K?(j \vee_{\mathrm{A}} b) \models_{\mathrm{S}} Kb$ (by *at least* presupposition)

On the other hand, from "Bob knows that Jackson is not on TV" we cannot conclude "Bob knows whether Bush or Ferrell is on TV"; this explains Schaffer's intuition of the contrast in difficulty between the two alternative questions:

(24) a. $K\neg j \not\models_{\mathrm{S}} K?(f \vee_a b)$

b. $K\neg j, K?(f \vee_a b) \not\models_{\mathrm{S}} Kb$

The following are two more interesting results we are now able to prove:

(25) $K?(a \vee_{\mathrm{A}} b) \;\equiv_{\mathrm{S}}\; Ka \vee Kb$

a. $K?(a \vee_{\mathrm{A}} b) \;\models_{\mathrm{S}}\; Ka \vee Kb$ (by *at least*)

b. $Ka \vee Kb \;\models_{\mathrm{S}}\; K?(a \vee_{\mathrm{A}} b)$ (by *at most* and symmetry)

(26) $K?(a \vee_{\mathrm{A}} b) \;\equiv_{\mathrm{S}}\; K(a \wedge \neg b) \vee K(b \wedge \neg a)$

a. $K?(a \vee_{\mathrm{A}} b) \;\models_{\mathrm{S}} K(a \wedge \neg b) \vee K(b \wedge \neg a)$ (by *exactly one*)

b. $K(a \wedge \neg b) \vee K(b \wedge \neg a) \;\models\; K?(a \vee_{\mathrm{A}} b)$

If we are ready to accommodate the exactly one presupposition, then "knowing whether A or B" is equivalent to "knowing (A and not B) or knowing (B and not A)". If we are ready to accommodate both presuppositions, then it is equivalent to "knowing A or knowing B". Under these circumstances, then, the various alternative notions of knowing-*wh* discussed in Section 2 turn out to be equivalent.

Should we then conclude that the semantic analyses of knowing whether A or B discussed there are correct after all? No. In a moment we will provide evidence that both the *symmetry* and the *exactly one* inferences are cancellable. A weaker or stronger semantic analysis would not be able to account for these cases. First, though, we add to the discussion a fourth possible candidate beyond the three alternative analyses introduced so far, namely Karttunen's (1977) influential theory of knowing *whether*:[5]

(27) a. Partition analysis (PA): $K?a \wedge K?b$
b. Weak analysis (WA): $Ka \vee Kb$
c. Strong analysis (SA): $K(a \wedge \neg b) \vee K(b \wedge \neg a)$
d. Karttunen (KA): $(a \rightarrow Ka) \wedge (b \rightarrow Kb)$

It is easy to prove that if we accommodate the *at least* and symmetry presuppositions, we have pragmatic equivalence between our partition theory and Karttunen's semantics:

(28) $K?(a \vee_{\mathrm{A}} b) \;\equiv_{\mathrm{S}}\; (a \rightarrow Ka) \wedge (b \rightarrow Kb)$
a. $K?(a \vee_{\mathrm{A}} b) \;\models\; (a \rightarrow Ka) \wedge (b \rightarrow Kb)$
b. $(a \rightarrow Ka) \wedge (b \rightarrow Kb) \;\models_{\mathrm{S}}\; K?(a \vee_{\mathrm{A}} b)$ (by *at least* and symmetry)

Each theory discussed above, apart from the partition theory, fails to account for cases of cancellation of whichever presuppositions are used to prove their pragmatic equivalence to our partition theory. Thus, KA fails to account for cases of cancellation of the *at least* or the symmetry presupposition; SA of the *at least* and *at most* presuppositions; and WA of all three presuppositions. We turn now to the cancellation cases.

6.3. Cancellations

Symmetry Presuppositions, like entailments, are not cancelable in positive sentences; the following sentence is contradictory:

(29) The king of France is bald, but there is no king of France.

Unlike entailed inferences, however, presuppositions can sometimes be cancelled under negation:

(30) a. It is not true that the king of France is bald, because there is no king of France.
b. ?The king of France is not bald, because there is no king of France.

[5] Actually, (27-d) is the so called simplified Karttunen analysis, see (Heim, 1994). (27-d) follows from Karttunen's analysis of alternative questions plus the clause: a knows Q iff a believes all the true answers to Q. Tucked away in footnote 11, p. 18, (Karttunen, 1977) proposes a more complex analysis which avoids some of the problems of the simplified version.

Our symmetry inference seems to follow the same pattern. Sentence (31) is contradictory, but (32-a) is acceptable.

(31) Bob knows whether Mary is Italian or French. Bob knows whether she is Italian, but he doesn't know whether she is French.

(32) a. It is not true that Bob knows whether Mary is Italian or French, because Bob knows whether Mary is Italian, but he doesn't know whether she is French.
b. ?Bob doesn't knows whether Mary is Italian or French, because Bob knows whether Mary is Italian, but he doesn't know whether she is French.

Example (32-a) is a clear case of presupposition cancellation and could be truly used in a situation where Bob knows that Mary is Italian, but he wonders whether she is French as well. Assume Mary is Italian and not French. Then the *exactly one* presupposition is satisfied in this case. SA gives us the right predictions then: (32-a) is true in this situation, and (31) is false. WA, and Karttunen's analysis, instead, which rely on symmetry, would give us the wrong predictions for these cases: (32-a) woud be predicted to be false in this situation, and (31) to be true.

Both constructions are maybe odd in this situation (because they violate symmetry) but still we seem to have semantic intuitions about them. Our partition semantics and SA allow us to explain these intuitions; a weaker semantic analysis would fail in these cases.

At least one. Consider the following sentence:

(33) It is not true that Bob knows whether Mary is Italian or French.

Suppose Mary is German. Then, according to KA, (33) is false irrespective to Bob's actual belief state.[6] According to SA and WA, instead, (33) is true, again, irrespective to Bob's actual belief state. This cannot be correct. Suppose Bob believes that Mary is French, then (33) is intuitively true, but if he knows that she is German, then (33) is intuitively false. For these cases, only PA gives us the right predictions.

At most one. Consider now the following sentence, an overt cancellation of the *at most* inference:

(34) Bob knows whether Mary is Italian, French or both.

[6] Karttunen himself was aware of this shortcoming of the simplified version of his theory. See footnote 5.

This example is problematic for any account like WA in which knowing whether A_1 or ... or A_n entails that at most one of the alternatives is true. Such an account predicts that (34) is contradictory, but it is not.

KA and PA give the correct analysis for this case. WA is again too weak. Suppose Mary is both Italian and French. Bob knows that she is Italian, but ignores that she is French. In this case, (34) is false, but WA predicts it to be true.

6.4. A stronger presupposition

In the remainder of this section we would like to discuss an alternative analysis of the presupposition of knowing whether A or B. Instead of the combination of the symmetry and the exactly one presuppositions, we could have assumed just one stronger presupposition:

(35) **Strong presupposition:**
- a. Knowing whether A or B *presupposes*
- b. Knowing (A or B) & knowing not (A & B)

By accommodation of only (35) we would have accounted for Schaffer's "true" judgment, just because the strong presupposition is stronger than the conjunction of the other two presuppositions.

(36) $K(a \vee b) \wedge K\neg(a \wedge b) \models (a \vee b) \wedge \neg(a \wedge b) \wedge K?a \leftrightarrow K?b$

But we have two arguments against assuming the strong presupposition. One is of an empirical nature, the other is more theoretical.

Empirical argument. Suppose you wonder whether Mary is French or Italian. You know she is one of the two, and you also know that Bob wrongly believes that she is either German or both Italian and French. Now consider the following sentence:

(37) Bob doesn't know whether Mary is French or Italian.

According to our theory your utterance would be felicitous here. Both the symmetry and the exactly one presuppositions are satisfied. According to the strong presupposition theory, instead, (37) is infelicitous in the described situation. Intuition are subtle here. But we believe the sentence is felicitous. The strong presupposition theory is too strong.

Theoretical argument. We have independent evidence for our three pragmatic inferences. All three can be related to more general principles ruling the use of disjunction. The symmetry presupposition can be seen as a particular case of Chemla's (2008) principle of epistemic similarity for disjunction, according to which a sentence $\phi(a \vee b)$ is felicitous provided the speaker believes $\phi(a)$ if and only if the speaker believes $\phi(b)$. The *at most* inference seems very similar to the exclusive readings of plain disjunction: these are generalized pragmatic effects, but still easy to cancel (cf. example (34)). Lastly, the *at least* inference relates to what (Zimmermann, 2000) refers to as exhaustive closure of possibilities triggered by *closed* disjunction uses. (Zimmermann, 2000, p. 267) distinguishes between closed and open disjunctions. The former end with a low phrase-final tone and claim to cover the space of all possibilities. *Open* disjunctions, instead, end on a high phrase-final tone and express the possibility of each disjunct without making any claim of completeness. Compare

(38) a. Do you want COFFEE or TEA [low phrase-final tone] ?
b. Do you want COFFEE or TEA [high phrase-final tone] ?

Only (38-a) triggers an *at least* inference, suggesting perhaps that such inference should not be encoded in the semantics of alternative questions, but possibly in the interpretation of the final falling contour.

To conclude, we have independent evidence for our three pragmatic inferences. The strong presupposition instead is hard to relate to more general principles. Furthermore assuming the strong presupposition would not leave space to account for the possibility that the symmetry presupposition is of a different nature to the other two pragmatic inferences. As the last discussion seems to suggest, it is possible that only the first is a genuine case of presupposition; the other two inferences might be just cases of generalized conversational effects.

7. Conclusion

Let us summarize the main results of this paper. We compared different possible analyses of *knowing whether* sentences and showed how to integrate the partitional analysis of alternative questions within epistemic logic. We examined different ways of making sense of the context-sensitivity of sentences of the form $K?(a \vee_{\mathrm{A}} b)$. As argued in section 4, a surprising fact about such sentences is that the same scenario can support the intuition that one and the same sentence can be true, false, or undefined in principle. However, we have seen that these judgments can be teased apart once we isolate the relevant parameters. By introducing a semantics of knowledge that distinguishes attention and relevance parameters, we saw that Schaffer's judgments can be derived without resorting to a special mechanism of accommodation of the alternatives raised by the question. Independently, we saw how judgments of undefinedness can be related in a precise way to the pragmatics of alternative questions. One thing we have not proposed here is a detailed comparison with the dynamic approach of the problem proposed in (Aloni and Égré, 2008), where the role of direct questions and the order in which they are posed was used to explain how the subject's attention can widen up. Likewise, we confined our logic to the case of a single agent, leaving open how to extend it to the multi-agent case. Both directions should be pursued in further work. Meanwhile, we hope to have established a fruitful connection between the broad issue of contextualism in epistemology on the one hand, and the articulation between attention and questions on the other.

Appendix

As an illustration of the system introduced in Section 3 we will prove one direction of (38-c):

(39) $K\ ?(a \vee_{\text{A}} b) \models K?a \wedge K?b$

Suppose $M, w, v \not\models_g K?a \wedge K?b$. This means either (a) or (b) is the case:

(40) (a) $M, w, v \not\models_g K?a$
(b) $M, w, v \not\models_g K?b$

Possibility (a) means that $\exists w' : wRw'$ such that $M, w', w \not\models_g ?a$. This means that:

(41) $M, w', w \models_g a \not\Leftrightarrow M, w, w \models_g a$

Which means:

(42) $w' \in V(a) \not\Leftrightarrow w \in V(a)$

which is equivalent to:

(43) $w' \in V(a)$ & $(V(a) = V(a)$ or $V(a) = V(b)) \not\Leftrightarrow$
$w \in V(a)$ & $(V(a) = V(a)$ or $V(a) = V(b))$

From (43) it follows that there is proposition $\alpha \in P$ such that:

(44) $w' \in \alpha$ & $(\alpha = V(a)$ or $\alpha = V(b)) \not\Leftrightarrow$
$w \in \alpha$ & $(\alpha = V(a)$ or $\alpha = V(b))$

Therefore, by the clauses for $p, a, =, \wedge$ and $\vee$ we can conclude that there is an $\alpha \in P$ such that:

(45) $M, w', w \models_{g[p/\alpha]} (p \wedge (p = a \vee p = b)) \not\Leftrightarrow$
$M, w, w \models_{g[p/\alpha]} (p \wedge (p = a \vee p = b))$

Which means, by the clause for ?, that:

(46) $M, w', w \not\models_g ?p\ (p \wedge (p = a \vee p = b))$

But, since wRw', this means that:

(47) $M, w, v \not\models_g K?p\ (p \wedge (p = a \vee p = b))$

Parallel reasoning for [b].

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