Pragmatic enrichments in state-based modal logic

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Introduction

- Grice's paradise: canonical divide between semantics and pragmatics
 - Pragmatic inference: derivable by conversational principles, cancellable, non-embeddable, ...
 - Semantic inference: not derivable by conversational principles, non-cancellable, embeddable, ...
- Gricean picture recently challenged by a class of modal inferences triggered by existential/disjunctive constructions:
 - Ignorance inference in epistemic indefinites and modified numerals
 - Free choice inferences in indefinites and disjunction
 - •
- Common core of these inferences:
 - Although derivable by conversational principles they lack other defining properties of pragmatic inferences
- Goal of this project: develop logics for such inferences which capture their hybrid behaviour by allowing pragmatics intrude into the recursive process of meaning composition
- ► Today: a logic for free choice (and ignorance) where the intruding pragmatic principle is a version of Grice's Maxim of Quality

Free choice (FC)

▶ FC inference: conjunctive meanings derived from disjunctive modal sentences contrary to the prescriptions of classical logic

$$(1) \qquad \diamondsuit(\alpha \lor \beta) \leadsto \diamondsuit\alpha \land \diamondsuit\beta \qquad (\mathsf{NB}: \diamondsuit\alpha \land \diamondsuit\beta \neq \diamondsuit(\alpha \land \beta))$$

- Classical examples:
 - (2) Deontic FC [Kamp 1973]
 - a. You may go to the beach or to the cinema.
 - b. \rightsquigarrow You may go to the beach and you may go to the cinema.
 - (3) Epistemic FC

[Zimmermann 2000]

- a. Mr. X might be in Victoria or in Brixton.
- b. \rightarrow Mr. X might be in Victoria and he might be in Brixton.

The paradox of free choice

Free choice permission in natural language:

(4) you may (A or B)
$$\rightsquigarrow$$
 you may A

But (5) not valid in standard deontic logic (von Wright 1968):

(5)
$$\diamond(\alpha \lor \beta) \to \diamond \alpha$$
 [Free Choice Principle]

- Plainly making the Free Choice Principle valid, for example by adding it as an axiom, would not do (Kamp 1973):
- ▶ The step leading to 2 in (6) uses the following valid principle:

(7) $\Diamond \alpha \rightarrow \Diamond (\alpha \lor \beta)$ [Modal Addition]

▶ Natural language counterpart of (7), however, seems invalid:

(8) you may A $\not\leadsto$ you may (A or B) [Ross's paradox]

 \Rightarrow Intuitions on natural language in direct opposition to the principles of classical logic

Reactions to paradox

- Paradox of Free Choice (FC) Permission:
 - $\begin{array}{cccc} (9) & 1. & \diamond a \\ & 2. & \diamond (a \lor b) \end{array}$

 - 3. *⇔b*
- Pragmatic solutions
 - FC inferences are conversational implicatures
 - \Rightarrow step leading to 3 is unjustified
- Semantic solutions
 - EC inferences are semantic entailments
 - \Rightarrow step leading to 3 is justified, but step leading to 2 is no longer valid
- Free choice: semantics or pragmatics?
- My proposal: a logic-based account beyond canonical semantics vs pragmatics divide
 - FC inference derived via 'pragmatic intrusion': $\Diamond (\alpha \lor \beta)^+ \models \Diamond \alpha$
 - (Modal) addition will fail but only for pragmatically enriched formulas
 - Upshot logic-based account: hybrid behaviour naturally derived

[assumption] [from 1, by modal addition] [from 2, by FC principle]

 $[\Rightarrow$ keep logic as is]

 \Rightarrow change the logic

Beyond Gricean paradise

| | | pragm. derivable | cancellable | non- embed. | proc. cost |
|--------------------|--|---------------------|-------------|----------------|---------------|
| Pra gma tics | Conversational implicature \overline{J} is always very punctual \rightsquigarrow J is not a good philosopher | + | + | + | high |
| Sem ant ics | <u>Classical entailment</u> I read some novels → I read something | _ | _ | - | low |
| 3rd Kind | $\frac{\text{FC inference}}{\text{You may do A } or B} \sim \\ \text{You may do } A$ | + | ? | ? | low* |
| | Scalar implicature I read some novels → I didn't read all novels | + | + | ? | high |

*Chemla and Bott. Processing inferences at the semantics/pragmatics frontier: disjunctions and free choice. *Cognition*, pages 380–396, 2014.

Argument against semantic accounts of FC

Free choice effects systematically disappear in negative contexts:

- (10) Dual Prohibition
 - a. You are not allowed to eat the cake or the ice-cream. \sim You are not allowed to eat either one

b. $\neg \diamondsuit(\alpha \lor \beta) \rightsquigarrow \neg \diamondsuit \alpha \land \neg \diamondsuit \beta$

c. $\neg \diamondsuit (\alpha \lor \beta) \neq \neg (\diamondsuit \alpha \land \diamondsuit \beta)$

Argument against pragmatic accounts of ${\rm FC}$

Free choice effects embeddable under universal quantification:

- (11) Universal FC
 - a. All of the boys may go to the beach or to the cinema. \rightsquigarrow All of the boys may go to the beach and all of the boys may go to the cinema.
 - b. $\forall x \diamondsuit (\alpha \lor \beta) \rightsquigarrow \forall x (\diamondsuit \alpha \land \diamondsuit \beta)$

Localist vs globalist view on implicatures

- ▶ (11) normally used to argue against globalist accounts of implicatures;
- But, localists (Fox et al) who predict the availability of embedded FC implicatures and therefore capture (11), need adjustments to capture (10).

(Alonso-Ovalle 2005)

(Chemla 2009)

Argument against most accounts (including localists view)

► Free choice effects also arise with wide scope disjunctions:

(12) Wide Scope FC

(Zimmermann 2000)

- a. Detectives may go by bus or they may go by boat. \rightsquigarrow Detectives may go by bus and may go by boat.
- b. Mr. X might be in Victoria or he might be in Brixton. \rightsquigarrow Mr. X might be in Victoria and might be in Brixton.
- $\mathsf{c.} \qquad \Diamond \alpha \lor \Diamond \beta \rightsquigarrow \Diamond \alpha \land \Diamond \beta$
- ▶ Wide scope FC hard to capture (not derivable by Gricean reasoning)
- ► Standard strategy: wide scope FC reduced to narrow scope FC:
 - (13) a. Detectives may go by bus or they may go by boat.
 - b. Logical Form: $\Diamond(\alpha \lor \beta)/\# \Diamond \alpha \lor \Diamond \beta$
- ▶ But then (14) would require dubious syntactic operations:
 - (14) a. You may email us or you can reach the Business License office at 949 644-3141. → You may email us
 - b. Logical Form: $\Diamond \alpha \lor \Diamond \beta / \# \Diamond (\alpha \lor \beta)$

(Simons' covert ATB movement would not work here, Alonso-Ovalle 2006)

Free choice: data and predictions

▶ FC inference in between semantics and pragmatics:

$$\begin{array}{lll} \text{(15)} & \text{a.} & \diamond(\alpha \lor \beta) \leadsto \diamond \alpha \land \diamond \beta & \qquad & & & & & & \\ \text{b.} & \neg \diamond(\alpha \lor \beta) \leadsto \neg \diamond \alpha \land \neg \diamond \beta & \qquad & & & & \\ \text{c.} & \forall x \diamond (\alpha \lor \beta) \leadsto \forall x (\diamond \alpha \land \diamond \beta) & \qquad & & & & & \\ \text{d.} & \diamond \alpha \lor \diamond \beta \leadsto \diamond \alpha \land \diamond \beta & \qquad & & & & & \\ \end{array}$$

| | N Scope FC | Dual Prohibition | Universal FC | W Scope FC |
|----------------|------------|-------------------------|--------------|------------|
| Semantic | yes | no | yes | ? |
| Pragm (global) | yes | yes | no | no |
| Pragm (local) | yes | no* | yes | no* |

Free choice: semantics or pragmatics?

- A purely semantic or pragmatic approach cannot account for this complex pattern of inferences
- I will present a hybrid approach where
 - ► FC inference derived by allowing pragmatic principles intrude in the recursive process of meaning composition
- ▶ On localist view: intruding EXH, a version of Quantity maxim;
- ► Here: intruding principle will be a version of Quality maxim.

A logic for pragmatic intrusion

- Pragmatic intrusion captured in a bilateral state-based modal logic which models assertion/rejection conditions rather than truth
- State-based modal logic: formulas are interpreted wrt info states (sets of possible worlds) rather than possible worlds
- Classical vs state-based modal logic
 - Classical modal logic:

$$M, w \models \phi$$
, where $w \in W$

State-based modal logic:

(support in info states)

 $[M = \langle W, R, V \rangle]$ (truth in worlds)

 $M, s \models \phi$, where $s \subseteq W$

Bilateral state-based modal logic:

 $\begin{array}{l} M,s\models\phi, \ ``\phi \text{ is assertable in }s"\\ M,s=\phi, \ ``\phi \text{ is rejectable in }s" \end{array}$

Partiality: although state-based logical consequence can be classical, we can have states where neither p nor ¬p is supported:

$$M, s \models p \quad \text{iff} \quad \forall w \in s : V(w, p) = 1$$
$$M, s \models \neg p \quad \text{iff} \quad \forall w \in s : V(w, p) = 0$$

 Info states: less determinate than worlds (comparable to truthmakers, situations, possibilities)

State-based modal logic: applications

Partial nature makes state-based systems particularly suitable to capture phenomena at the semantics-pragmatics interface

Epistemic contradictions

- (16) #It might be raining and it is not raining. (Veltman, Yalcin)
 - Challenge: $\Diamond p \land \neg p \models \bot$, while $\Diamond p \not\models p$

Pragmatic enrichments

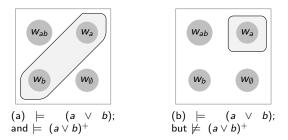
- Free choice: you may do A or $B \Rightarrow$ you may do A (today's focus)
- Ignorance triggered by or and at least (vOrmondt & MA):

| (17) | a. | ?I have two or three children. | (Grice) |
|------|----|--------------------------------------|-------------------|
| | b. | ?I have at least two children. | (Nouwen & Geurts) |
| (18) | a. | Every woman in my family has two o | r three children. |
| | b. | Every woman in my family has at leas | st two children. |

Crucial ingredient: *split disjunction* from team logic (Y&V, H&S-T)

State-based modal logic: split disjunction

- A state s supports φ ∨ ψ iff s can be split into two substates, each supporting one of the disjuncts;
- A state s supports (φ ∨ ψ)⁺ iff s can be split into two non-empty substates, each supporting one of the disjuncts;



- ▶ Pragmatically enriched disjunctions convey ignorance wrt which disjunct is true: (a ∨ b)⁺ no longer supported by state on the right;
- Pragmatic enrichment function ⁺ defined in terms of NE (also from team logic).

State-based modal logic: NE

- Conversation is ruled by a principle that prescribes to avoid contradictions ('avoid ⊥') [follows from QUALITY]
- \blacktriangleright $_{\rm NE}$ is the formal counterpart of 'avoid \perp ' in state-based systems
- ▶ In classical logic no non-trivial way to model 'avoid \perp ': $\neg \bot = \top$
- In a state-based semantics:
 - Ø → state of logical insanity, supports everything including contradictions: Ø ⊨ p ∧ ¬p
 - NE models 'avoid ⊥' by requiring the supporting state to be non-empty (≠ Ø)

 $M, s \models \text{NE} \quad \text{iff} \quad s \neq \emptyset$ $M, s \models \text{NE} \quad \text{iff} \quad s = \emptyset$

Core of the proposal: ignorance and FC inferences follow from systematic "intrusion" of 'avoid ⊥' (= NE) into the recursive process of meaning composition

State-based modal logic: pragmatic enrichment Implementation

Pragmatically enriched formulas φ⁺ come with the requirement to satisfy NE ('avoid ⊥') distributed along each of their subformulas:

$$egin{array}{rcl} m{p}^+ &=& m{p} \wedge \mathrm{NE} \ (
eg \phi)^+ &=&
eg \phi^+ \wedge \mathrm{NE} \ (\phi ee \psi)^+ &=& (\phi^+ \wedge \mathrm{NE}) ee (\psi^+ \wedge \mathrm{NE}) \end{array}$$

Main results

- By pragmatically enriching every formula, we derive:
 - Narrow scope FC: $\Diamond (\alpha \lor \beta)^+ \models \Diamond \alpha \land \Diamond \beta$
 - Wide scope FC: $(\Diamond \alpha \lor \Diamond \beta)^+ \models \Diamond \alpha \land \Diamond \beta$ (with restrictions)
 - Universal FC: $\forall x \diamond (\alpha \lor \beta)^+ \models \forall x (\diamond \alpha \land \diamond \beta)$
 - Distribution: $\forall x (\alpha \lor \beta)^+ \models \exists x \alpha \land \exists x \beta$ and more
- while no undesirable side effects obtain with other configurations:
 - Dual prohibition: $\neg \Diamond (a \lor b)^+ \models \neg \Diamond a \land \neg \Diamond b$
- ▶ Subtle predictions wrt wide scope FC confirmed by pilot experiment
- ► Cognitively plausible: natural to assume that speakers disregard Ø in ordinary conversations

Bilateral State-Based Modal Logic (BSML) Language

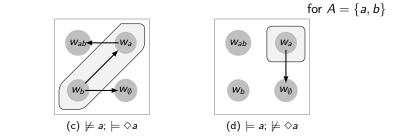
$$\phi \quad := \quad \boldsymbol{p} \mid \neg \phi \mid \phi \land \phi \mid \phi \lor \phi \mid \diamondsuit \phi \mid \mathsf{NE}$$

where $p \in A$.

Models and States

- Classical Kripke models: $M = \langle W, R, V \rangle$
- States: $s \subseteq W$, sets of worlds in a Kripke model

Examples



Semantic clauses

$$[M = \langle W, R, V \rangle; s, t, t' \subseteq W]$$

$$\begin{array}{lll} M,s\models p & \text{iff} & \forall w\in s: V(w,p)=1\\ M,s\models p & \text{iff} & \forall w\in s: V(w,p)=0\\ M,s\models \neg\phi & \text{iff} & M,s\models \phi\\ M,s\models \neg\phi & \text{iff} & M,s\models \phi\\ M,s\models \phi\wedge\psi & \text{iff} & M,s\models \phi\&M,s\models\psi\\ M,s\models \phi\wedge\psi & \text{iff} & \exists t,t':t\cup t'=s\&M,t\models \phi\&M,t'=\psi\\ M,s\models \phi\vee\psi & \text{iff} & \exists t,t':t\cup t'=s\&M,t\models \phi\&M,t'\models\psi\\ M,s\models \phi\vee\psi & \text{iff} & M,s\models \phi\&M,s=\psi\\ M,s\models \phi\vee\psi & \text{iff} & M,s\models \phi\&M,s=\psi\\ M,s\models \diamond\phi & \text{iff} & \forall w\in s:\exists t\subseteq R^{\rightarrow}(w):t\neq\emptyset\&t\models\phi\\ M,s\models \otimes\phi & \text{iff} & \forall w\in s:R^{\rightarrow}(w)=\phi\\ M,s\models \otimese & \text{iff} & s\neq\emptyset\\ M,s\models \otimese & \text{iff} & s=\emptyset\\ \end{array}$$

where $R^{\rightarrow}(w) = \{v \mid wRv\}$

Pragmatic intrusion

$$p^{+} = p \wedge \text{NE}$$

$$(\neg \phi)^{+} = \neg \phi^{+} \wedge \text{NE}$$

$$(\phi \lor \psi)^{+} = (\phi^{+} \wedge \text{NE}) \lor (\psi^{+} \wedge \text{NE})$$

$$(\phi \land \psi)^{+} = (\phi^{+} \wedge \text{NE}) \land (\psi^{+} \wedge \text{NE})$$

$$(\Diamond \phi)^{+} = \Diamond \phi^{+} \wedge \text{NE}$$

$$\text{NE}^{+} = \text{NE}$$

Logical consequence

•
$$\phi \models \psi$$
 iff for all $M, s : M, s \models \phi \Rightarrow M, s \models \psi$

 $\blacktriangleright \phi \models_X \psi \text{ iff for all } (M, s) \in X : M, s \models \phi \ \Rightarrow \ M, s \models \psi$

State-sensitive constraints on accessibility relation

▶ *R* is indisputable in (*M*, *s*) iff $\forall w, v \in s : R^{\rightarrow}(w) = R^{\rightarrow}(v)$

▶ *R* is state-based in (*M*, *s*) iff $\forall w \in s : R^{\rightarrow}(w) = s$ where $R^{\rightarrow}(w) = \{v \mid wRv\}$

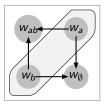
Main ingredients: constraints on accessibility relation

State-sensitive constraints on accessibility relation:

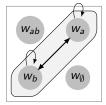
- R is indisputable in (M, s) iff ∀w, v ∈ s : R→(w) = R→(v) → all worlds in s access exactly the same set of worlds
- ► *R* is state-based in (*M*, *s*) iff $\forall w \in s : R^{\rightarrow}(w) = s$

 \mapsto all and only worlds in s are accessible within s

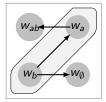
where $R^{\rightarrow}(w) = \{v \mid wRv\}$



(e) indisputable



(f) state-base (and so also indisputable)



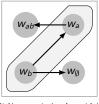
(g) neither

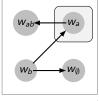
- Difference deontic vs epistemic modals captured by different properties of accessibility relation:
 - Epistemics: R is state-based
 - Deontics: *R* is possibly indisputable

(e.g. in performative uses)

Main ingredients: split disjunction

- ► A bilateral version of split disjunction from team logic:
 - A state s supports φ ∨ ψ iff s can be split into two substates, each supporting one of the disjuncts;
 - A state *s* rejects $\phi \lor \psi$ iff *s* rejects ϕ and rejects ψ .
- Pragmatically enriched disjunction:
 - After pragmatic intrusion: $(\phi \lor \psi)^+ =: (\phi^+ \land \text{NE}) \lor (\psi^+ \land \text{NE})$
 - A state s supports (φ ∨ ψ)⁺ iff s can be split into two non-empty substates, each supporting one of the disjuncts, e.g.





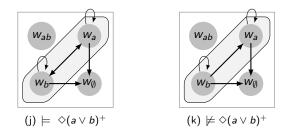
 $(\mathsf{h})\models a \lor b; \models (a \lor b)^+$

(i) $\models a \lor b; \not\models (a \lor b)^+$

▶ Pragmatic enrichment vacuous under negation: $\neg(a \lor b)^+ = \neg((a \land \text{NE}) \lor (b \land \text{NE})) = \neg(a \land \text{NE}) \land \neg(b \land \text{NE}) =$ $(\neg a \lor \neg \text{NE}) \land (\neg b \lor \neg \text{NE}) = \neg a \land \neg b = \neg(a \lor b)$

Main ingredients: modals

- ► A "classical" notion of modality:
 - A state s supports ◊φ iff for all w ∈ s: there is a non-empty subset of the set of worlds accessible from w which support φ
 - A state s rejects ◊ φ iff for all w ∈ s: the set of worlds accessible from w rejects φ
- \Rightarrow Free choice effect derived in combination with enriched disjunctions



Suppose s supports ◊a but not ◊b ⇒ no b-world accessible from some w in s ⇒ (a ∨ b)⁺ not supported by any subset of worlds accessible from w ⇒ ◊(a ∨ b)⁺ not supported in s

Results propositional BSML

Before pragmatic intrusion

► The NE-free fragment of BSML is equivalent to classical modal logic (CML): $\phi \models_{BSML} \psi$ iff $\phi \models_{CML} \psi$ (ϕ, ψ are NE-free)

But we can capture infelicity of epistemic contradictions by putting constraints on epistemic accessibility relation:

- 1. Epistemic contradiction: $\Diamond a \land \neg a \models \bot (= \neg \text{NE})$ (if *R* is state-based)
- 2. Non-factivity: $\diamond a \not\models a$

After pragmatic intrusion

- FC (and ignorance) inferences derived for pragmatically enriched disjunction:
 - Narrow scope FC: $\diamond (a \lor b)^+ \models \diamond a \land \diamond b$
 - ▶ Wide scope FC: $(\Diamond a \lor \Diamond b)^+ \models \Diamond a \land \Diamond b$ (if *R* is indisputable)
 - Ignorance: $(a \lor b)^+ \models \Diamond a \land \Diamond b$

(if *R* is indisputable) (if *R* is state-based)

- Only disjunctions in positive environments (and logically equivalent formulas) affected by pragmatic intrusion:
 - Dual prohibition: $\neg \Diamond (a \lor b)^+ \models \neg \Diamond a \land \neg \Diamond b$

Applications: epistemic contradiction

Epistemic contradiction and non-factuality

1. $\diamond a \land \neg a \models \bot$

[if R is state-based]

2. **∂*a* |≠ a

Epistemics vs deontics

- Differ wrt properties of accessibility relation:
 - Epistemics: *R* is state-based
 - ► Deontics: *R* is possibly indisputable (e.g. in performative uses)
- Epistemic contradiction predicted for epistemics, but not for deontics:
 - (19) #It might be raining and it is not raining.
 - (20) You don't smoke but you may smoke.

Applications: epistemic free choice

Narrow scope and wide scope ${\rm FC}$

1.
$$\diamond (a \lor b)^+ \models \diamond a \land \diamond b$$

2.
$$(\Diamond a \lor \Diamond b)^+ \models \Diamond a \land \Diamond b$$

[if R is indisputable]

Epistemic modals

- ► *R* is state-based, therefore always indisputable:
 - (21) He might either be in London or in Paris. [+fc, narrow]
 - (22) He might be in London or he might be in Paris. [+fc, wide]
- \blacktriangleright \Rightarrow narrow and wide scope $_{\rm FC}$ always predicted for pragmatically enriched epistemics
- Working hypothesis on cancellation:
 - 1. +-enrichment cancellable only in very special circumstances (with cancellation involving high processing costs);
 - 2. some expressions (e.g. epistemic indefinites, modified numerals) trigger obligatory +-enrichments.

Applications: deontic free choice

Narrow scope and wide scope ${\rm FC}$

1.
$$\diamond (a \lor b)^+ \models \diamond a \land \diamond b$$

2.
$$(\Diamond a \lor \Diamond b)^+ \models \Diamond a \land \Diamond b$$

[if R is indisputable]

Deontic modals

- R may be indisputable if speaker is knowledgable (e.g. in performative uses)
- Predictions:
 - \blacktriangleright \Rightarrow narrow scope $_{\rm FC}$ always predicted for enriched deontics
 - $\blacktriangleright \Rightarrow$ wide scope $_{\rm FC}$ only if speaker knows what is permitted/obligatory
- ► Further consequence: all cases of (overt) FC cancellations can be taken to involve a wide scope configuration (rather than more costly cancellation of +-enrichment)

Deontic FC: comparison with localist view

Our proposal vs Fox (2007)

| | NS+K | $NS \neg K$ | WS+K | WS¬K |
|------------|------|-------------|------|------|
| MA | yes | yes | yes | no |
| Fox (2007) | yes | no | no | no |

 $\mathsf{K}\mapsto\mathsf{speaker}\xspace$ knows what is permitted/obligatory;

 $\mathsf{NS} \mapsto \mathsf{narrow} \text{ scope } \mathsf{FC}; \mathsf{WS} \mapsto \mathsf{wide} \text{ scope } \mathsf{FC}.$

- Our predictions confirmed by pilot experiment (Cremers et al. 2017)
- Speaker knowledge has effect on FC inference only in wide scope configurations:
 - (23) We may either eat the cake or the ice-cream. [narrow, +fc]
 - (24) Either we may eat the cake or the ice-cream. [wide, +/-fc]

Position of *either* favors a narrow scope interpretation in (23), while it forces a wide scope interpretation in (24) (Larson 1985)

Deontic FC: comparison with localist view

- ► Recent localist accounts predict wide scope FC for cases paraphrasable with "or else".
 - (25) We may eat the cake or (else) we may eat the ice-cream.
- ► The generalisation that all wide scope FC cases are paraphrasable by "or else" is not correct. (26) seems to have FC interpretations:
 - (26) Either we may eat the cake or (# else) the ice-cream.
- And also: "or else"-cases do not seem to necessarily give rise to FC reading:
 - (27) You may eat the cake or else (you may eat) the ice-cream. I don't know which.
- ► Finally, even if the generalisation were correct, an uniform account of narrow and wide scope FC inferences seems preferable.

Deontic FC: (overt) FC cancellations

- Prediction: all natural cases of overt FC cancellations involve a wide scope configuration
- Sluicing in (28) arguably triggers wide scope configuration (Fusco 2018):
 - (28) You may either eat the cake or the ice-cream, I don't know which (you may eat). [wide, -fc]
- ▶ Cf. with (29) where sluicing triggers a narrow scope configuration:
 - (29) You may either eat the cake or the ice-cream, I don't care which (you eat). [narrow, +fc]
- ▶ Wide scope configuration also plausible for (30) (Kaufmann 2016):
 - (30) You may either eat the cake or the ice-cream, it depends on what John has taken. [wide, -fc]

Conclusions

- Free choice and ignorance: a mismatch between logic and language
- Grice's insight:
 - stronger meanings can be derived using general principles of conversation
- Standard implementation: two separate components
 - Semantics: classical logic
 - Pragmatics: Gricean reasoning

Elegant, but incorrect

- My proposal: a logic for pragmatic intrusion
 - Free choice (and ignorance) derived by letting pragmatics intrude into semantic composition;
 - Relevant intruding pragmatic principle:

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avoid \perp = \text{NE} \mapsto \text{ignore } \emptyset (cognitively plausible)
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- Classical logic can be recovered (as NE-free fragment);
- Bilateral state-based logic defines assertion/rejection conditions rather than truth.

Appendix: Ignorance

- ▶ Plain disjunctions give rise to ignorance effects (Gazdar 1976):
 - (31) a. John has two or three children. → speaker doesn't know how many
 - b. $\alpha \lor \beta \rightsquigarrow \Diamond \alpha \land \Diamond \beta$ [epistemic \Diamond]
- Strong effect: cancellable only in special circumstances
 - (32) ??I have two or three children.
 - (33) I have two or three children. Guess how many!
- Cross-linguistic evidence. In languages lacking explicit or, disjunctive meaning expressed by adding a suffix/particle expressing uncertainty to the main verb:
 - (34) Johnš Billš v?aawuumšaa.
 John-nom Bill-nom 3-come-pl-fut-infer
 'John or Bill will come'
 - (35) Johnš Billš v?aawuum.
 John-nom Bill-nom 3-come-pl-fut
 'John and Bill will come'

[Maricopa, Gil 1991, p. 102]

Obviation

- Ignorance effects obviated under universals:
 - (36) ??One of my sisters has two or three children.
 - (37) Every woman in my family has two or three children. \sim Some woman has two and some woman has three
- Similar effects with modified numerals (Nouwen *et al.*):
 - (38) a. ??I have at least two children.
 - b. ??One of my sisters has at least two children.
 - c. Every woman in my family has at least two children.
- And epistemic indefinites (Kratzer & Shimoyama, Aloni & Port):
 - (39) a. Irgendjemand hat aangerufen. #Rate mal wer! IRGEND-someone called. #Guess who!
 - Jeder Junge hat irgendjemanden geküsst. Rate mal wen J hat geküsst!
 Every boy kissed IRGEND-someone. Guess who J kissed!

Distribution

- ► Conjunctive uses of *or* under universals (Spector, Fox, Klinedinst):
 - (40) Distribution
 - a. Every woman in my family has two or three children. \rightsquigarrow some woman has two and some woman has three
 - b. $\forall x(\alpha \lor \beta) \rightsquigarrow \exists x\alpha \land \exists x\beta$
- Distribution pragmatically derived via negations of universal alternatives: ∀x(α ∨ β) + ¬∀xα + ¬∀xβ ⊨ ∃xα ∧ ∃xβ
- But distributive inferences may obtain in the absence of plain negated universal inferences (Crnic, Chemla, Fox 2015):
 - (41) Every brother of mine has been married to a woman or a man. $\rightsquigarrow \exists x \alpha \land \exists x \beta$, even when $\not \rightsquigarrow \neg \forall x \alpha$
- Different behaviour \exists and \diamond surprising for pragmatic accounts:

(42) a.
$$\forall x (\alpha \lor \beta) \rightsquigarrow \exists x \alpha \land \exists x \beta$$
 [distribution]
b. $\exists x (\alpha \lor \beta) \not \rightsquigarrow \exists x \alpha \land \exists x \beta$

Summary desiderata

Free choice

44) a.
$$\diamond(\alpha \lor \beta) \rightsquigarrow \diamond \alpha \land \diamond \beta$$

b. $\diamond \alpha \lor \diamond \beta \rightsquigarrow \diamond \alpha \land \diamond \beta$
c. $\neg \diamond(\alpha \lor \beta) \rightsquigarrow \neg \diamond \alpha \land \neg \diamond \beta$
d. $\forall x \diamond (\alpha \lor \beta) \rightsquigarrow \forall x (\diamond \alpha \land \diamond \beta)$

[narrow scope FC] [wide scope FC] [dual prohibition] [universal FC]

Ignorance

| (45) | a. | $\alpha \lor \beta \rightsquigarrow \Diamond \alpha \land \Diamond \beta$ | [ignorance1] |
|------|----|--|----------------|
| | b. | $\exists x (\alpha \lor \beta) \rightsquigarrow \exists x (\diamond \alpha \land \diamond \beta)$ | [ignorance2] |
| | c. | $\forall x(\alpha \lor \beta) \rightsquigarrow \exists x \alpha \land \exists x \beta$ | [distribution] |
| | d. | $\forall x (\alpha \lor \beta) \not \rightsquigarrow \forall x (\diamond \alpha \land \diamond \beta)$ | [obviation] |

So far no unified account to this complex pattern of inference;
 Next: a logic-based account where all these inferences will follow as "reasonable inferences" by combining a split notion of disjunction with a state-based mechanism of pragmatic intrusion.

[...] an inference is reasonable just in case, in every context in which the premisses could appropriately be asserted, it is impossible for anyone to accept the premisses without committing himself to the conclusion [Stalnaker 1975]

Logic of Pragmatic Intrusion: first order

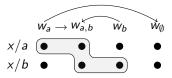
Language

$$\phi \quad := \quad P x_1, \dots, x_n \mid \neg \phi \mid \phi \land \phi \mid \phi \lor \phi \mid \Diamond \phi \mid \forall x \phi \mid \text{Ne}$$

Models and States

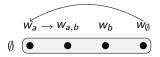
- First order models: $M = \langle W, R, D, I \rangle$
- First order states: sets of world-assignment pairs

Example of a first order state

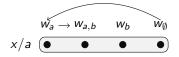


Operations on first order states (Dekker 1993)

► A state *s* with empty assignment:

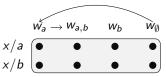


• An individual x-extension of s, s[x/a]:



• The universal x-extension of s, $\bigcup_{d \in D} s[x/d]$:

 $D = \{a, b\}$



Quantifiers

- $s \models \forall x \phi$ iff the universal *x*-extension of *s* supports ϕ
- ▶ $s = \forall x \phi$ iff there is an individual *x*-extension of *s* which rejects ϕ
- $s \models \exists x \phi$ iff there is an individual *x*-extension of *s* which supports ϕ
- ▶ $s = \exists x \phi$ iff the universal *x*-extension of *s* rejects ϕ

Examples $D = \{a, b\}$ WPaPb WP2 WPb Wa Ø • $\not\models \exists x P x : \not\models \forall x P x : \neq \exists x P x : \neq \forall x P x$ WPb WP2 WPaPb Wa Ø • $\models \exists x P x: \not\models \forall x P x: \not= \exists x P x: \not= \forall x P x$ WPaPb Wa WP2 WPb (\bullet) Ø • $\models \exists x P x : \models \forall x P x : \neq \exists x P x : \neq \forall x P x$ WP2 WPb WPaPb Wa Ø

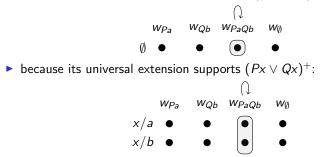
Obviation

$$\blacktriangleright \forall x (\alpha \lor \beta)^+ \not\models \forall x (\diamond \alpha \land \diamond \beta)$$

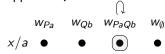
[R state-based]

Counterexample

State of maximal information supports $\forall x (Px \lor Qx)^+$:

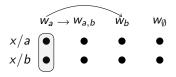


But does not support ∀x(◊Px ∧ ◊Qx), because its universal extension does not support (◊Px ∧ ◊Qx) because for example the following does not support ◊Qx:

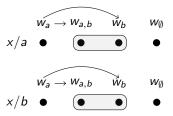


Modals and quantifiers: Quantifying in

- When interpreting ◊φ/□φ in s, for each i ∈ s we evaluate φ wrt a state constructed by combining g_i with worlds accessible from w_i
- Eg., to evaluate $\Box Px$ in the following state:



• We need to evaluate *Px* in the following two states:



As a result, variables behave classically as rigid designators

Bilateral state-based modal logic: first order extension

Language

$$\phi \quad := \quad P x_1, \ldots, x_n \mid \neg \phi \mid \phi \lor \phi \mid \Diamond \phi \mid \forall x \phi \mid \text{NE}$$

where $x_1, \ldots, x_n \in V$, $P \in A$.

Models

► M = ⟨W, R, D, I⟩, where W is a set of worlds, R is an accessibility relation, D a set of individuals and I is a world-dependent interpretation function for A

States: sets of world-assignments pairs

- An index is a pair $i = \langle w_i, g_i \rangle$ where $w_i \in W$ is a world and $g_i : V \to D$ is a partial assignment function;
- ▶ A state *s* is a set of indices s.t. for all $i, j \in s$: $dom(g_i) = dom(g_j)$.

Semantic clauses

| $M, s \models Px_1, \ldots, x_n$ | iff | $\forall i \in s : \langle g_i(x_1), \dots, g_i(x_n) \rangle \in I(w_i)(P)$ |
|---|-----|--|
| $M, s \models Px_1, \ldots, x_n$ | iff | $\forall i \in s : \langle g_i(x_1), \dots, g_i(x_n) \rangle \notin I(w_i)(P)$ |
| $M, s \models \neg \phi$ | iff | $M, s = \phi$ |
| $M, s = \neg \phi$ | iff | $\textit{M}, \textit{s} \models \phi$ |
| $\textit{M}, \textit{s} \models \phi \lor \psi$ | iff | $\exists t, t': t \cup t' = s \& M, t \models \phi \& M, t' \models \psi$ |
| $\textit{\textit{M}},\textit{\textit{s}} \rightleftharpoons \phi \lor \psi$ | iff | $M, s \models \phi \& M, s \models \psi$ |
| $M, s \models \Diamond \phi$ | iff | $\forall i \in s : \exists X \subseteq R^{\rightarrow}(w_i) : X \neq \emptyset \& X[g_i] \models \phi$ |
| $M, s = \diamondsuit \phi$ | iff | $orall i \in {m s}: R^{ ightarrow}({m w}_i)[g_i] = \mid \phi$ |
| $M, s \models \forall x \varphi$ | iff | $M, \bigcup_{d \in D} s[x/d] \models \varphi$ |
| $M, s = \forall x \varphi$ | iff | $M, s[x/d] \models arphi, 	ext{ for some } d \in D$ |
| $M, s \models$ ne | iff | $s eq \emptyset$ |
| M, s = NE | iff | $s = \emptyset$ |
| | | |

where $X[g_i] = \{ \langle w, g_i \rangle \mid w \in X \}$ & $s[x/d] = \{ \langle w, g[x/d] \rangle \mid \langle w, g \rangle \in s \}.$

Logical consequence

•
$$\phi \models \psi$$
 iff for all $M: M, s \models \phi \Rightarrow M, s \models \psi$

Pragmatic intrusion

$$p^{+} = p \land \text{NE}$$

$$(\neg \phi)^{+} = \neg \phi^{+} \land \text{NE}$$

$$(\phi \lor \psi)^{+} = (\phi^{+} \land \text{NE}) \lor (\psi^{+} \land \text{NE})$$

$$(\forall x \phi)^{+} = \forall x (\phi^{+} \land \text{NE})$$

$$(\diamondsuit \phi)^{+} = \diamondsuit \phi^{+} \land \text{NE}$$

$$\text{NE}^{+} = \text{NE}$$

Auxiliary notation

- ▶ $\bot =: \neg NE$
- $\blacktriangleright \phi \land \psi =: \neg (\neg \phi \lor \neg \psi)$
- $\blacktriangleright \ \Box \phi =: \neg \diamondsuit \neg \phi$
- $\blacktriangleright \exists x\phi =: \neg \forall x \neg \phi$
- $(\phi \lor_+ \psi) =: (\phi \lor \psi)^+$

Summary of results first order extension

Before pragmatic intrusion

The NE-free fragment is equivalent to classical quantified modal logic

After pragmatic intrusion

- Ignorance2, universal FC and distribution derived for pragmatically enriched disjunction:
 - ▶ Ignorance2: $\exists x(\alpha \lor \beta)^+ \models \exists x(\Diamond \alpha \land \Diamond \beta)$

[R is state-based]

- Universal FC: $\forall x \diamondsuit (\alpha \lor \beta)^+ \models \forall x (\diamondsuit \alpha \land \diamondsuit \beta)$
- Distribution: $\forall x (\alpha \lor \beta)^+ \models \exists x \alpha \land \exists x \beta$