Pragmatic enrichments in state-based modal logic

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State-based modal logic

$[M = \langle W, R, V \rangle]$

Formulas evaluated wrt info states rather than possible worlds

- Classical modal logic: (truth in worlds)

$$M, w \models \phi, \text{ where } w \in W$$

- State-based modal logic: (support in info states)

$$M, s \models \phi, \text{ where } s \subseteq W$$

- Partial nature: although state-based logical consequence can be classical, we can have states where neither p nor $\neg p$ is supported:

$$\begin{array}{rcl} M, s \models p & \text{iff} & \forall w \in s : V(w, p) = 1 \\ M, s \models \neg p & \text{iff} & \forall w \in s : V(w, p) = 0 \end{array}$$

- Info states: less determinate than worlds, just like
  - truthmakers, situations, possibilities, ...
- Technically:
  - Truthmakers, possibilities, ...: points in a partially ordered set
  - Info states: sets of worlds (also elements of a partially ordered set)

State-based modal logic: applications

Partial nature makes state-based systems particularly suitable to capture phenomena at the semantics-pragmatics interface

Epistemic contradictions

(1) #It might be raining and it is not raining. (Veltman, Yalcin)

- Challenge: $\Diamond p \wedge \neg p \models \bot$, while $\Diamond p \not\models p$
- Two ways to capture (1): (i) via *state-sensitive* constraint on epistemic accessibility relation assuming a classical notion of modality; (ii) in non-modal fragment with *might* $p \mapsto p^+ \vee \top$.

Pragmatic enrichments

- Free choice: you may do A or $B \Rightarrow$ you may do A (today's focus)
- Ignorance triggered by *or* and *at least* (vOrmondt & MA):

(2) a. ?I have two or three children. (Grice)
b. ?I have at least two children. (Nouwen & Geurts)

(3) a. Every woman in my family has two or three children.
b. Every woman in my family has at least two children.

Crucial ingredient: *split disjunction* from team logic (Y&V, H&S-T)

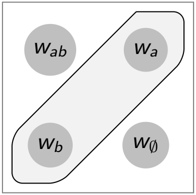
Three notions of $\vee$ in state-based systems (Aloni 2016)

$M, s \models \phi \vee_1 \psi$	iff	$\forall w \in s : M, \{w\} \models \phi$ or $M, \{w\} \models \psi$	(classical disjunction)
$M, s \models \phi \vee_2 \psi$	iff	$\exists t, t' : t \cup t' = s$ & $M, t \models \phi$ & $M, t' \models \psi$	(split disjunction)
$M, s \models \phi \vee_3 \psi$	iff	$M, s \models \phi$ or $M, s \models \psi$	(inquisitive/truthmaker disjunction)

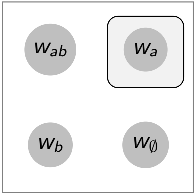
- $\vee_1$ and $\vee_2$ equivalent in distributive/flat systems where they behave classically, while $\vee_3$ leads to violation of LEM;

Today

- $\vee_2$ in a non-distributive system;
- Source of non-distributivity: pragmatic enrichment function $^+$ defined in terms of NE (also from team logic).



(a) $\models (a \vee_{1/2/*3} \neg a)$;
and $\models (a \vee_2 \neg a)^+$



(b) $\models (a \vee_{1/2/3} \neg a)$;
but $\not\models (a \vee_2 \neg a)^+$

Structure of the talk

1. Motivation: the paradox of free choice
2. Bilateral state-based modal logic (BSML)
   - Definitions
   - Results
3. Bilateral state-based logic (BSL) (non-modal fragment)
   - Some motivation
   - Axiomatisation (building on Y&V, 2017[1])
4. Conclusion
5. Appendix: linguistic applications

Outlook

- BSML: combination of the following
  1. $\neg$: negation from truthmakers semantics [$\mapsto$ bilateralism]
  2. $\vee$: (bilateral) split disjunction from team logic
  3. $\Diamond$: (bilateral) "classical" modality from possibility semantics
  4. NE: from team logic [gives us pragmatic enrichments]
- NE is the only source of non-classical behaviour:
  - NE-free fragment of BSML = classical modal logic

[1] Fan Yang and Jouko Väänänen (2017). Propositional team logics. *Annals of Pure and Applied Logic*. 168(7): 1406–1441

The challenge of free choice (FC)

- Classical examples of FC inferences:

(4) Deontic FC inference [Kamp 1973]

a. You may go to the beach *or* to the cinema.

b. $\rightsquigarrow$ You may go to the beach *and* you may go to the cinema.

(5) Epistemic FC inference [Zimmermann 2000]

a. Mr. X might be in Victoria *or* in Brixton.

b. $\rightsquigarrow$ Mr. X might be in Victoria *and* he might be in Brixton.

- Logical rendering of FC inferences:

(6) $\Diamond(\alpha \vee \beta) \rightsquigarrow \Diamond\alpha \wedge \Diamond\beta$ (NB: $\Diamond\alpha \wedge \Diamond\beta \neq \Diamond(\alpha \wedge \beta)$)

Is this inference valid in classical modal logic? No.

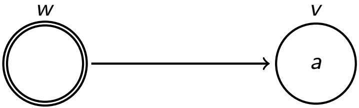


Figure: $M, w \models \Diamond(a \vee b)$, but $M, w \not\models \Diamond a \wedge \Diamond b$

The paradox of free choice

- Free choice permission in natural language:

(7) You may (A or B) $\rightsquigarrow$ You may A

- But (8) not valid in standard deontic logic (von Wright 1968):

(8) $\Diamond(\alpha \vee \beta) \to \Diamond\alpha$ [Free Choice Principle]

- Plainly making the Free Choice Principle valid, for example by adding it as an axiom, would not do (Kamp 1973):

(9)
1. $\Diamond a$ [assumption]
2. $\Diamond(a \vee b)$ [from 1, by classical reasoning]
3. $\Diamond b$ [from 2, by free choice principle]

- The step leading to 2 in (9) uses the classically valid (10):

(10) $\Diamond\alpha \to \Diamond(\alpha \vee \beta)$ [Modal Addition]

- Natural language counterpart of (10), however, seems invalid:

(11) You may A $\not\rightsquigarrow$ You may (A or B) [Ross's paradox]

$\Rightarrow$ Intuitions on natural language in direct opposition to the principles of classical logic

Reactions to paradox

- Paradox of Free Choice (FC) Permission:

(12)

1.	$\Diamond a$	[assumption]
2.	$\Diamond(a \vee b)$	[from 1, by modal addition]
3.	$\Diamond b$	[from 2, by FC principle]

- Pragmatic solutions [$\Rightarrow$ keep logic as is]
  - FC inferences are conversational implicatures, i.e. pragmatic inferences derived as the product of rational interactions between cooperative language users (+ classical logic meanings)

  $\Rightarrow$ step leading to 3 is unjustified
- Semantic solutions [$\Rightarrow$ change the logic]
  - FC inferences are semantic entailments

  $\Rightarrow$ step leading to 3 is justified, but step leading to 2 is no longer valid
- Free choice: semantics or pragmatics? My view:
  - FC inferences: neither purely semantic nor purely pragmatic
  - derivable by conversational principles but lacking other defining properties of gricean inferences
- Proposal: a logic-based account of FC inferences beyond canonical semantics vs pragmatics divide

Argument against semantic accounts of FC

Free choice effects systematically disappear in negative contexts:

(13) Dual Prohibition (Alonso-Ovalle 2005)

a. You are not allowed to eat the cake or the ice-cream.
$\rightsquigarrow$ You are not allowed to eat either one.

b. $\neg\Diamond(\alpha \vee \beta) \rightsquigarrow \neg\Diamond\alpha \wedge \neg\Diamond\beta$

Argument against pragmatic accounts of FC

Free choice effects embeddable under universal quantification:

(14) Universal FC (Chemla 2009)

a. All of the boys may go to the beach or to the cinema.
$\rightsquigarrow$ All of the boys may go to the beach and all of the boys may go to the cinema.

b. $\forall x \Diamond(\alpha \vee \beta) \rightsquigarrow \forall x(\Diamond\alpha \wedge \Diamond\beta)$

Argument against most accounts (including localist view)

Free choice effects also arise with wide scope disjunctions:

(15) Wide Scope FC (Zimmermann 2000)

a. Mr. X might be in Victoria or he might be in Brixton. $\rightsquigarrow$ Mr. X might be in Victoria and might be in Brixton.

b. $\Diamond\alpha \vee \Diamond\beta \rightsquigarrow \Diamond\alpha \wedge \Diamond\beta$

Free choice: summary data and predictions

(16) a. $\Diamond(\alpha \vee \beta) \rightsquigarrow \Diamond\alpha \wedge \Diamond\beta$ [Narrow Scope FC]

b. $\neg\Diamond(\alpha \vee \beta) \rightsquigarrow \neg\Diamond\alpha \wedge \neg\Diamond\beta$ [Dual Prohibition]

c. $\forall x\Diamond(\alpha \vee \beta) \rightsquigarrow \forall x(\Diamond\alpha \wedge \Diamond\beta)$ [Universal FC]

d. $\Diamond\alpha \vee \Diamond\beta \rightsquigarrow \Diamond\alpha \wedge \Diamond\beta$ [Wide Scope FC]

	N Scope FC	Dual Prohibition	Universal FC	W Scope FC
Semantic	yes	no	yes	no*
Pragmatic	yes	yes	no	no

Free choice: semantics or pragmatics?

- A purely semantic or pragmatic approach cannot account for this complex pattern of inference
- I propose a hybrid approach where
    - FC inference derived by allowing “pragmatics” to intrude in the recursive process of meaning composition
- Pragmatic intrusion captured in a bilateral state-based modal logic which models assertion/rejection conditions rather than truth

Bilateral state-based modal logic

- Classical modal logic: (truth in worlds)

$$M, w \models \phi, \text{ where } w \in W$$

- State-based modal logic: (support in info states)

$$M, s \models \phi, \text{ where } s \subseteq W$$

- Bilateral state-based modal logic:

$$\begin{array}{ll} M, s \models \phi, \text{ “}\phi \text{ is assertable in } s\text{”}, & \text{with } s \subseteq W \\ M, s \mathrel{=\!\!|} \phi, \text{ “}\phi \text{ is rejectable in } s\text{”}, & \text{with } s \subseteq W \end{array}$$

$[M = \langle W, R, V \rangle]$

Pragmatic intrusion in state-based modal logic

- Conversation is ruled by a principle that prescribes to avoid contradictions ('avoid $\bot$') [follows from QUALITY]
- Proposal: FC inferences follow from the systematic "intrusion" of 'avoid $\bot$' into the recursive process of meaning composition

Implementation

To model such intrusion we need a way to formally represent 'avoid $\bot$':

- In classical logic no non-trivial way to do it: $\neg\bot = \top$

In a state-based semantics:

- $\emptyset \mapsto$ state of logical insanity, supports everything including contradictions: $\emptyset \models p \wedge \neg p$
- But then we can represent 'avoid $\bot$' by means of a constant, NE, which requires the supporting state to be non-empty ($\neq \emptyset$)

$$M, s \models \text{NE} \quad \text{iff} \quad s \neq \emptyset$$
$$M, s \mathrel{=\mkern-3mu|} \text{NE} \quad \text{iff} \quad s = \emptyset$$

Pragmatic intrusion in state-based modal logic

Pragmatic enrichment

- Pragmatically enriched formulas ϕ^+ come with the requirement to satisfy NE ('avoid $\bot$') distributed along each of their subformulas:

$$\begin{array}{rcl} p^+ & = & p \wedge \text{NE} \\ (\neg\phi)^+ & = & \neg\phi^+ \wedge \text{NE} \\ (\phi \vee \psi)^+ & = & (\phi^+ \wedge \text{NE}) \vee (\psi^+ \wedge \text{NE}) \\ & \dots & \end{array}$$

Main result

- By pragmatically enriching every formula, we derive:
  - Narrow scope FC: $\Diamond(\alpha \vee \beta)^+ \models \Diamond\alpha \wedge \Diamond\beta$
  - Wide scope FC: $(\Diamond\alpha \vee \Diamond\beta)^+ \models \Diamond\alpha \wedge \Diamond\beta$ (with restrictions)
  - Universal FC: $\forall x\Diamond(\alpha \vee \beta)^+ \models \forall x(\Diamond\alpha \wedge \Diamond\beta)$
  - Distribution: $\forall x(\alpha \vee \beta)^+ \models \exists x\alpha \wedge \exists x\beta$ and more
- while no undesirable side effects obtain with other configurations:
  - Dual prohibition: $\neg\Diamond(a \vee b)^+ \models \neg\Diamond a \wedge \neg\Diamond b$
- Subtle predictions wrt wide scope FC confirmed by pilot experiment
- Cognitively plausible: natural to assume that speakers disregard $\emptyset$ in ordinary conversations

Bilateral State-Based Modal Logic (BSML)

Language

$$\phi \quad := \quad p \mid \neg\phi \mid \phi \wedge \phi \mid \phi \vee \phi \mid \Diamond\phi \mid \mathrm{NE}$$

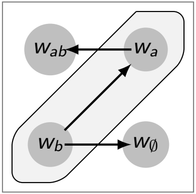
where $p \in A$.

Models and States

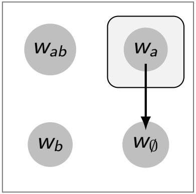
- Classical Kripke models: $M = \langle W, R, V \rangle$
- States: $s \subseteq W$, sets of worlds in a Kripke model

Examples

for $A = \{a, b\}$



(a) $\not\models a$; $\models \Diamond a$



(b) $\models a$; $\not\models \Diamond a$

Semantic clauses

$[M = \langle W, R, V\rangle;\ s, t, t' \subseteq W]$

$$
\begin{array}{rcl}
M, s \models p & \text{iff} & \forall w \in s : V(w, p) = 1 \\
M, s \dashv p & \text{iff} & \forall w \in s : V(w, p) = 0 \\
M, s \models \neg\phi & \text{iff} & M, s \dashv \phi \\
M, s \dashv \neg\phi & \text{iff} & M, s \models \phi \\
M, s \models \phi \wedge \psi & \text{iff} & M, s \models \phi \ \&\ M, s \models \psi \\
M, s \dashv \phi \wedge \psi & \text{iff} & \exists t, t' : t \cup t' = s \ \&\ M, t \dashv \phi \ \&\ M, t' \dashv \psi \\
M, s \models \phi \vee \psi & \text{iff} & \exists t, t' : t \cup t' = s \ \&\ M, t \models \phi \ \&\ M, t' \models \psi \\
M, s \dashv \phi \vee \psi & \text{iff} & M, s \dashv \phi \ \&\ M, s \dashv \psi \\
M, s \models \Diamond\phi & \text{iff} & \forall w \in s : \exists t \subseteq R^{\rightarrow}(w) : t \neq \emptyset \ \&\ t \models \phi \\
M, s \dashv \Diamond\phi & \text{iff} & \forall w \in s : R^{\rightarrow}(w) \dashv \phi \\
M, s \models \mathrm{NE} & \text{iff} & s \neq \emptyset \\
M, s \dashv \mathrm{NE} & \text{iff} & s = \emptyset
\end{array}
$$

where $R^{\rightarrow}(w) = \{v \mid wRv\}$

Translations into Modal Information Logic (vBenthem 18)

- Bilateral state-based logic (no $\Diamond$ and no $\textsc{ne}$)

$$
\begin{aligned}
(\neg\phi)^+ &= (\phi)^- \\
(\neg\phi)^- &= (\phi)^+ \\
(\phi\wedge\psi)^+ &= (\phi)^+ \wedge (\psi)^+ \\
(\phi\wedge\psi)^- &= \langle \mathit{sup}\rangle(\phi)^-(\psi)^- \\
(\phi\vee\psi)^+ &= \langle \mathit{sup}\rangle(\phi)^+(\psi)^+ \\
(\phi\vee\psi)^- &= (\phi)^- \wedge (\psi)^-
\end{aligned}
$$

- Truthmaker semantics

$$
\begin{aligned}
(\neg\phi)^+ &= (\phi)^- \\
(\neg\phi)^- &= (\phi)^+ \\
(\phi\wedge\psi)^+ &= \langle \mathit{sup}\rangle(\phi)^+(\psi)^+ \\
(\phi\wedge\psi)^- &= (\phi)^- \vee (\psi)^- \\
(\phi\vee\psi)^+ &= (\phi)^+ \vee (\psi)^+ \\
(\phi\vee\psi)^- &= \langle \mathit{sup}\rangle(\phi)^-(\psi)^-
\end{aligned}
$$

Pragmatic intrusion

$$
\begin{aligned}
p^+ &= p \wedge \mathrm{NE} \\
(\neg\phi)^+ &= \neg\phi^+ \wedge \mathrm{NE} \\
(\phi \vee \psi)^+ &= (\phi^+ \wedge \mathrm{NE}) \vee (\psi^+ \wedge \mathrm{NE}) \\
(\phi \wedge \psi)^+ &= (\phi^+ \wedge \mathrm{NE}) \wedge (\psi^+ \wedge \mathrm{NE}) \\
(\Diamond\phi)^+ &= \Diamond\phi^+ \wedge \mathrm{NE} \\
\mathrm{NE}^+ &= \mathrm{NE}
\end{aligned}
$$

Logical consequence

- $\phi \models \psi$ iff for all $M, s : M, s \models \phi \;\Rightarrow\; M, s \models \psi$
- $\phi \models_X \psi$ iff for all $(M, s) \in X : M, s \models \phi \;\Rightarrow\; M, s \models \psi$

State-sensitive constraints on accessibility relation

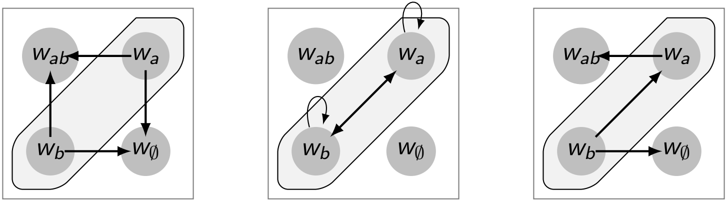
- R is indisputable in (M, s) iff $\forall w, v \in s : R^{\rightarrow}(w) = R^{\rightarrow}(v)$
- R is state-based in (M, s) iff $\forall w \in s : R^{\rightarrow}(w) = s$

where $R^{\rightarrow}(w) = \{v \mid wRv\}$

Main ingredients: constraints on accessibility relation

- State-sensitive constraints on accessibility relation:
  - R is indisputable in (M, s) iff $\forall w, v \in s : R^{\rightarrow}(w) = R^{\rightarrow}(v)$
    $\mapsto$ all worlds in s access exactly the same set of worlds
  - R is state-based in (M, s) iff $\forall w \in s : R^{\rightarrow}(w) = s$
    $\mapsto$ all and only worlds in s are accessible within s

  where $R^{\rightarrow}(w) = \{v \mid wRv\}$



(c) indisputable

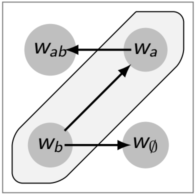
(d) state-base (and so also indisputable)

(e) neither

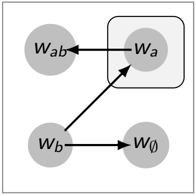
- Difference deontic vs epistemic modals captured by different properties of accessibility relation:
  - Epistemics: R is state-based
  - Deontics: R is possibly indisputable (e.g. in performative uses)

Main ingredients: split disjunction

- A bilateral version of split disjunction from team logic:
  - A state s supports $\phi \vee \psi$ iff s can be split into two substates, each supporting one of the disjuncts;
  - A state s rejects $\phi \vee \psi$ iff s rejects ϕ and rejects ψ.
- Pragmatically enriched disjunction:
  - After pragmatic intrusion: $(\phi \vee \psi)^+ =: (\phi^+ \wedge \mathrm{NE}) \vee (\psi^+ \wedge \mathrm{NE})$
  - A state s supports $(\phi \vee \psi)^+$ iff s can be split into two non-empty substates, each supporting one of the disjuncts, e.g.



(f)$\models a \vee b$; $\models (a \vee b)^+$



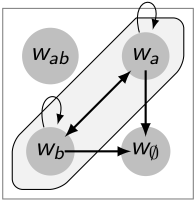
(g)$\models a \vee b$; $\not\models (a \vee b)^+$

  - Pragmatic enrichment vacuous under negation: $\neg(a \vee b)^+ = \neg((a \wedge \mathrm{NE}) \vee (b \wedge \mathrm{NE})) = \neg(a \wedge \mathrm{NE}) \wedge \neg(b \wedge \mathrm{NE}) = (\neg a \ \vee \neg\mathrm{NE}) \wedge (\neg b \ \vee \neg\mathrm{NE}) = \neg a \wedge \neg b = \neg(a \vee b)$

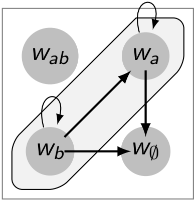
Main ingredients: modals

- A "classical" notion of modality:
  - A state s supports $\Diamond\phi$ iff for all $w \in s$: there is a non-empty subset of the set of worlds accessible from w which support ϕ
  - A state s rejects $\Diamond\phi$ iff for all $w \in s$: the set of worlds accessible from w rejects ϕ

$\Rightarrow$ Free choice effect derived in combination with enriched disjunctions



(h) $\models \Diamond(a \vee b)^+$



(i) $\not\models \Diamond(a \vee b)^+$

- Suppose s supports $\Diamond a$ but not $\Diamond b$ $\Rightarrow$ no b-world accessible from some w in s $\Rightarrow$ $(a \vee b)^+$ not supported by any subset of worlds accessible from w $\Rightarrow$ $\Diamond(a \vee b)^+$ not supported in s

Results propositional BSML

Before pragmatic intrusion

- The NE-free fragment of BSML is equivalent to classical modal logic (CML): $\phi \models_{BSML} \psi$ iff $\phi \models_{CML} \psi$ (ϕ, ψ are NE-free)
- But we can capture infelicity of epistemic contradictions by putting constraints on epistemic accessibility relation:
  1. Epistemic contradiction: $\Diamond a \wedge \neg a \models \bot (= \neg \mathrm{NE})$ (if R is state-based)
  2. Non-factivity: $\Diamond a \not\models a$

After pragmatic intrusion

- FC (and ignorance) inferences derived for pragmatically enriched disjunction:
  - Narrow scope FC: $\Diamond(a \vee b)^+ \models \Diamond a \wedge \Diamond b$
  - Wide scope FC: $(\Diamond a \vee \Diamond b)^+ \models \Diamond a \wedge \Diamond b$ (if R is indisputable)
  - Ignorance: $(a \vee b)^+ \models \Diamond a \wedge \Diamond b$ (if R is state-based)
- Only disjunctions in positive environments (and logically equivalent formulas) affected by pragmatic intrusion:
  - Dual prohibition: $\neg\Diamond(a \vee b)^+ \models \neg\Diamond a \wedge \neg\Diamond b$

Modal Definability: preliminaries

Frames

- Classical frame: $F = \langle W, R \rangle$
- Model based on frame: $M_F = \langle W_F, R_F, V \rangle$ for some V
- Frame validity: $F, s \models \phi$ iff for all M_F: $M_F, s \models \phi$
- (F, s) is indisputable/state based if R_F is indisputable/state-based wrt s

Disjoint Union Closure Property (del Valle-Inclan 2019)

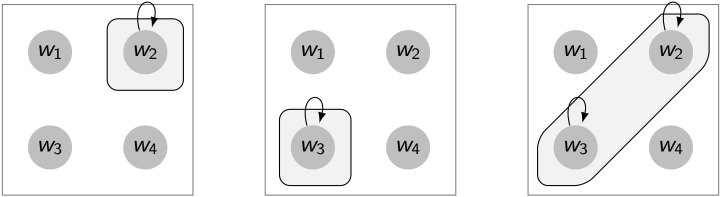
- $M_1, s_1 \models \phi$ and $M_2, s_2 \models \phi \Rightarrow M_1 \sqcup M_2, s_1 \sqcup s_2 \models \phi$
- $F_1, s_1 \models \phi$ and $F_2, s_2 \models \phi \Rightarrow F_1 \sqcup F_2, s_1 \sqcup s_2 \models \phi$

Modal Definability: negative result

Modal Definability (del Valle-Inclan 2019)

The class of state-based/indisputable (F, s) is **not definable**, i.e. there is no formula ϕ such that $F, s \models \phi$ iff (F, s) is state-based/indisputable.

Proof: Suppose there were such a ϕ. Then for state-based/indisputable (F_1, s_1) and (F_2, s_2): $F_1, s_1 \models \phi$ and $F_2, s_2 \models \phi$. By Disjoint Union Closure Property, $F_1 \sqcup F_2, s_1 \sqcup s_2 \models \phi$, but $R_{1\sqcup 2}$ need not be state-based/indisputable wrt $s_1 \sqcup s_2$.



(j) (F_1, s_1) (k) (F_2, s_2) (l) $(F_1 \sqcup F_2, s_1 \sqcup s_2)$

Modal Definability: conjectures

- Conjecture 1: indisputability/state-based "definable" by rules:
  - Indisputability:

$$\frac{\Diamond A^+ \vee \Diamond B}{\Diamond A^+} \text{ (wide scope FC)}$$

  - State-based property:

$$\frac{\Diamond\neg A \wedge A}{\bot} \text{ (epistemic contradiction (EC))}$$

  where rule Y defines model property X if $\phi \models_X \psi$ iff $\phi \vdash_{+Y} \psi$

- If we add (EC) to an axiomatisation of classical ML we would lose classical *reductio* (Aloni, Incurvati, Schlöder 2019[2]).

$$\cfrac{\cfrac{\cfrac{\Diamond\neg A \qquad [A]^1}{\Diamond\neg A \wedge A}\ (\wedge\text{-Introduction})}{\bot}\ \text{(epistemic contradiction)}}{\neg A}\ (\text{classical } \textit{reductio})^1$$

- Conjecture 2: properly adapted version of AIS proof-system sound and complete with respect to class of state-based (F, s) (if we only consider NE-free fragment).

[2] Aloni, Incurvati, Schlöder. Weak assertion meets information states. APA 2019

Non-modal fragment: Bilateral State-based Logic (BSL)

Language BSL

$$\phi \quad := \quad p \mid \neg\phi \mid \phi \wedge \phi \mid \phi \vee \phi \mid \text{NE}$$

where $p \in A$.

Motivations: epistemic modals

- $\text{MIGHT } \phi := (\phi \wedge \text{NE}) \vee \top$ (where $\top := p \vee \neg p$)
- Epistemic contradictions: $p \wedge \text{MIGHT } \neg p \models \text{NE} \wedge \neg \text{ NE}$ ($\neq \neg\text{NE}$)
- Ignorance: $(a \vee b)^+ \models \text{MIGHT } a \wedge \text{MIGHT } b$
- Epistemic FC: $\text{MIGHT } (a \vee b)^+ \models \text{MIGHT } a \wedge \text{MIGHT } b$
- Under negation:
  - $\neg(a \vee b)^+ \models \neg a \wedge \neg b$ ("non-modal" dual prohibition)
  - $\neg \text{ MIGHT } \phi \models \neg \text{ NE}$

    $\Rightarrow$ prediction: linguistic *m*ight never scopes under negation

Tautologies and contradictions

- $\top := p \vee \neg p$ — always supported
- $\bot := \mathrm{NE} \wedge \neg\mathrm{NE}$ — never supported
- NE — supported by all non-empty states
- $\neg\mathrm{NE}$ — supported only by empty state

Effect of negation

$$
\begin{tikzcd}
\top \arrow[ddrr, "\neg" description] & & \bot \arrow[ddll] \\
& & \\
\mathrm{NE} \arrow[rr, leftrightarrow, "\neg" description] & & \neg\ \mathrm{NE}
\end{tikzcd}
$$

Failure of substitution under $\neg$: $\neg\top \equiv \neg\ \mathrm{NE}$ but $\neg\neg\top \not\equiv \neg\neg\ \mathrm{NE}$

Axiomatisation of BSL

- Observation: BSL = CPL$^+$ + (non Boolean) negation

Classical Propositional Logic$^+$ (CPL$^+$)

- CPL$^+$ (Y&V 2017):
  - Language L^- : $\phi := p \mid \neg p \mid \phi \wedge \phi \mid \phi \vee \phi \mid \mathrm{NE} \mid \neg\mathrm{NE}$
  - Model Theory: our support clauses for $p, \phi \wedge \phi, \phi \vee \phi, \mathrm{NE}$, with clauses for $\neg p$, $\neg\mathrm{NE}$ equivalent to our anti-support clauses for p, NE.
  - Proof Theory: PT(CPL$^+$)
- **Soundness and Completeness CPL$^+$** (Y&V, 2017)
  $\phi \models \psi \Leftrightarrow \phi \vdash_{PT(CPL^+)} \psi$ for $\phi, \psi \in L^-$

Plan

- Extend PT(CPL$^+$) with sound rules for negation and then prove completeness of BSL via translation using Lemma*
- Lemma*: Every $\phi \in L$ can be translated into a $\phi^- \in L^-$ such that $\phi \vdash \phi^-$ and $\phi^- \vdash \phi$.

Proof-theory (rules for negation)

Double Negation

$$(\neg\neg_1)\ \frac{\neg\neg A}{A} \qquad (\neg\neg_2)\ \frac{A}{\neg\neg A}$$

De Morgan Laws

$$(DM_1)\ \frac{\neg(A \vee B)}{\neg A \wedge \neg B} \qquad (DM_2)\ \frac{\neg(A \wedge B)}{\neg A \vee \neg B}$$

$$(DM_3)\ \frac{\neg A \wedge \neg B}{\neg(A \vee B)} \qquad (DM_4)\ \frac{\neg A \vee \neg B}{\neg(A \wedge B)}$$

Proof-theory (from Y&V)

Atomic excluded middle

$$(\mathsf{EM}_0)\ \frac{}{p \vee \neg p}$$

Conjunction (classical introduction and elimination rules)

$$(\wedge\mathsf{I})\ \frac{A \qquad B}{(A \wedge B)} \qquad (\wedge\mathsf{E})\ \frac{(A \wedge B)}{A} \qquad (\wedge\mathsf{E})\ \frac{(A \wedge B)}{B}$$

Proof-theory (from Y&V)

Disjunction (weak introduction and elimination rules)

$$(\vee \mathsf{I}^-)\ \frac{A}{A \vee B} \text{ if } B \text{ is } \textsc{ne}\text{-free} \qquad (\vee \mathsf{I}^-)\ \frac{B}{A \vee B} \text{ if } A \text{ is } \textsc{ne}\text{-free}$$

$$(\vee \mathsf{E}^-)\ \frac{(A \vee B) \qquad \begin{array}{c}[A]\\ D_1\\ C\end{array} \qquad \begin{array}{c}[B]\\ D_2\\ C^3\end{array}}{C} \text{ if undischarged assumptions in } D_1, D_2 \text{ are } \textsc{ne}\text{-free}^4$$

[3] No restriction on C needed because we have union closure property (we don't have Boolean disjunction).

[4] NE-free condition here corresponds to classical formula condition in Y&V because we do not have inquisitive disjunction and $\neg$NE is provably equivalent to classical $p \wedge \neg p$.

Proof-theory (from Y&V)

Disjunction (weakening, commutative and associative laws)

$$(\mathrm{W}\vee)\ \frac{A}{A \vee A} \qquad (\mathrm{Com}\vee)\ \frac{A \vee B}{B \vee A} \qquad (\mathrm{Ass}\vee)\ \frac{A \vee (B \vee C)}{(A \vee B) \vee C}$$

Disjunction (weak substitution)

$$(\vee\mathrm{Sub}^{-})\ \frac{(A \vee B) \qquad \begin{array}{c}[B] \\ D_0 \\ C\end{array}}{(A \vee C)}$$ if undischarged assumptions in D_0 are NE-free.

Proof-theory (from Y&V)

Weak contradiction ($\neg\mathrm{NE} = \bot_w$)

$$\bot_w\mathrm{I}\ \frac{p \wedge \neg p}{\neg\mathrm{NE}} \qquad \bot_w\mathrm{E}\ \frac{A \vee \neg\mathrm{NE}}{A}$$

Strong contradiction ($\bot = \mathrm{NE} \wedge \neg\mathrm{NE}$)

$$\bot\mathrm{I}\ \frac{\bigvee_{i\in s}(\pi_1^i p_1 \wedge \cdots \wedge \pi_n^i p_n \wedge\ \mathrm{NE}) \wedge \bigvee_{j\in t}(\pi_1^j p_1 \wedge \cdots \wedge \pi_n^j p_n \wedge\ \mathrm{NE})}{\mathrm{NE} \wedge \neg\mathrm{NE}}\ (\text{if } s \neq t)$$

$$\bot\mathrm{E}\ \frac{\mathrm{NE} \wedge \neg\mathrm{NE}}{A} \qquad \bot\mathrm{Ctr}\ \frac{A \vee (\mathrm{NE} \wedge \neg\ \mathrm{NE})}{\mathrm{NE} \wedge \neg\mathrm{NE}}$$

[$\pi_m^i = \neg$ or blank, depending on value of p_m in $i \in s$]

...

Proof-theory (from Y&V)

Strong Elimination Rules

$$
(\mathrm{SE}_1)\ \dfrac{A \qquad \begin{array}{c}[A[\phi_{s_1}/NE_m]] \\ D_1 \\ C\end{array} \qquad \begin{array}{c}[A[\phi_{s_k}/NE_m]] \\ D_k \\ C\end{array}}{C}
$$

where ϕ_s is the formula in disjunctive normal form fully characterising team s; $\{s_1, \ldots s_k\}$ is the set of all non-empty teams on a set of indices N; and NE_m is a subformula of A occurring at position m

$$
(\mathrm{SE}_2)\ \dfrac{A \qquad \begin{array}{c}[A[\psi \wedge \neg NE/\psi_m]] \\ D_1 \\ C\end{array} \qquad \begin{array}{c}[A[\psi \wedge NE/\psi_m]] \\ D_2 \\ C\end{array}}{C}
$$

Soundness and completeness for BSL

- Soundness: $\phi \vdash \psi \ \Rightarrow \ \phi \models \psi$
  Proof: it is enough to show that double negation and de Morgan rules are sound (easy induction).
- Completeness: $\phi \models \psi \ \Rightarrow \ \phi \vdash \psi$
  Proof: $\phi \models \psi \Rightarrow \phi^- \models \psi^-$ (soundness and lemma*) $\Rightarrow \phi^- \vdash \psi^-$ (completeness CPL$^+$) $\Rightarrow \psi^- \vdash \psi$ (lemma*) $\Rightarrow \phi \vdash \phi^-$ (lemma*) $\Rightarrow$ $\phi \vdash \psi$
  - Lemma*: Every $\phi \in L$ can be translated into a $\phi^- \in L^-$ such that $\phi \vdash \phi^-$ and $\phi^- \vdash \phi$.
    Proof: next page
  - Completeness CPL$^+$: $\phi \models \psi \Rightarrow \phi \vdash_{PT(CPL^+)} \psi$ for $\phi, \psi \in L^-$
    Proof: see Y&V 2017.

Soundness and completeness for BSL

Lemma*: Every $\phi \in L$ can be translated into a $\phi^- \in L^-$ such that $\phi \vdash \phi^-$ and $\phi^- \vdash \phi$.

Proof: Define ϕ^- as follows:

$$\begin{array}{rcl}
p^- & = & p \\
\mathrm{NE}^- & = & \mathrm{NE} \\
(\psi \vee \chi)^- & = & \psi^- \vee \chi^- \\
(\psi \wedge \chi)^- & = & \psi^- \wedge \chi^- \\
(\neg\psi)^- & = & \neg\psi, \text{ if } \psi = p, \mathrm{NE} \\
 & = & \chi, \text{ if } \psi = \neg\chi \\
 & = & \neg\phi_1^- \vee \neg\phi_2^- \text{ if } \psi = \phi_1 \wedge \phi_2 \\
 & = & \neg\phi_1^- \wedge \neg\phi_2^- \text{ if } \psi = \phi_1 \vee \phi_2
\end{array}$$

Induction on ϕ. Base: trivial; $\phi = \psi \vee \chi$ (uses HI and $\vee$Sub$^-$); $\phi = \psi \wedge \chi$ (uses HI, $\wedge$I. and $\wedge$E); $\phi = \neg\psi$ (uses $\neg\neg$ and DM rules).

Conclusions

- Free choice: a mismatch between logic and language
- Grice's insight:
  - stronger meanings can be derived using general principles of conversation
- Standard implementation: two separate components
  - Semantics: classical logic
  - Pragmatics: Gricean reasoning

  Elegant picture, but incorrect for free choice
- My proposal: a state-based modal logic for pragmatic intrusion
  - Free choice derived by letting pragmatic principles intrude into semantic composition;
  - Classical logic can be recovered (as NE-free fragment);
  - Adopted bilateral system defines assertion/rejection conditions rather than truth.
- Future research:
  - Logic: proof-theory (modal extension); syntactic (via NE) vs semantic (via elimination of empty state) characterisation of pragmatic intrusion
  - Language: testing of predictions (experimental); analysis of overt FC cancellations (theoretical)

Applications: epistemic contradiction

Epistemic contradiction and non-factuality

1. $\Diamond a \wedge \neg a \models \bot$ [if R is state-based]
2. $\Diamond a \not\models a$

Epistemics vs deontics

- Differ wrt properties of accessibility relation:
  - Epistemics: R is state-based
  - Deontics: R is possibly indisputable (e.g. in performative uses)
- Epistemic contradiction predicted for epistemics, but not for deontics:

(17) #It might be raining and it is not raining. (Veltman, Yalcin)

(18) You don't smoke but you may smoke.

Applications: epistemic free choice

Narrow scope and wide scope FC

1. $\Diamond(a \vee b)^+ \models \Diamond a \wedge \Diamond b$
2. $(\Diamond a \vee \Diamond b)^+ \models \Diamond a \wedge \Diamond b$ [if R is indisputable]

Epistemic modals

- R is state-based, therefore always indisputable:

(19) He might either be in London or in Paris. [+fc, narrow]

(20) He might be in London or he might be in Paris. [+fc, wide]

- $\Rightarrow$ narrow and wide scope FC always predicted for pragmatically enriched epistemics

Applications: deontic free choice

Narrow scope and wide scope FC

1. $\Diamond(a \vee b)^+ \models \Diamond a \wedge \Diamond b$
2. $(\Diamond a \vee \Diamond b)^+ \models \Diamond a \wedge \Diamond b$ [if R is indisputable]

Deontic modals

- R may be indisputable if speaker is knowledgable (e.g. in performative uses)
- Predictions:
  - $\Rightarrow$ narrow scope FC always predicted for enriched deontics
  - $\Rightarrow$ wide scope FC only if speaker knows what is permitted/obligatory
- Further consequence: all cases of (overt) FC cancellations involve a wide scope configuration

Deontic FC: comparison with localist view

- Current proposal vs Fox (2007)

	NS+K	NS¬K	WS+K	WS¬K
MA	yes	yes	yes	no
Fox (2007)	yes	no	no	no

K $\mapsto$ speaker knows what is permitted/obligatory;

NS $\mapsto$ narrow scope FC; WS $\mapsto$ wide scope FC.

- MA's predictions confirmed by pilot experiment (Cremers et al. 2017)
- Speaker knowledge has effect on FC inference only in wide scope configurations:

(21) We may either eat the cake or the ice-cream. [narrow, +fc]

(22) Either we may eat the cake or the ice-cream. [wide, +/–fc]

Position of *either* favors a narrow scope interpretation in (21), while it forces a wide scope interpretation in (22) (Larson 1985)

Deontic FC: (overt) FC cancellations

- Prediction: all cases of (overt) FC cancellations involve a wide scope configuration
- Sluicing arguably triggers wide scope configuration in (23) but not in (24) (Fusco 2018):

(23) You may either eat the cake or the ice-cream, I don't know which (you may eat). [wide, –fc]

(24) You may either eat the cake or the ice-cream, I don't care which (you eat). [narrow, +fc]

- Wide scope configuration also plausible for (25) (Kaufmann 2016):

(25) You may either eat the cake or the ice-cream, it depends on what John has taken. [wide, –fc]