Weak assertion meets information states: a logic for epistemic modality and quantification

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Introduction

Goal

A fully worked out quantified logic for epistemic modality which

- derives the infelicity of epistemic contradictions
- solves puzzles arising from the combination of epistemic modals and quantifiers

while staying as close as possible to classical logic.

Outlook

1. Two challenges
   - Infelicity of epistemic contradictions
   - Quantification in situation of partial information
2. Quantified Epistemic Multilateral Logic (QEML)
   - Motivation for multilateralism
   - Proof theory and model theory
   - Soundness and completeness & first order classicality
3. Applications and discussion
   - Epistemic contradictions & non-factivity of $\Diamond$
   - Epistemic modals and quantifiers
   - The reach of classicality: embedded cases of epistemic contradictions

Infelicity of epistemic contradictions

- Epistemic modal *might* can be used to form Moore-like sentences (Wittgenstein, Veltman):

(1) #It's raining and I don't believe that it is raining.
(2) #It's raining and it might not be raining.

- But only (2) also infelicitous in embedded contexts (Yalcin 2007):

(3) Suppose that it's raining and I don't believe that it is raining.
(4) #Suppose that it's raining and it might not be raining.

⇒ A purely pragmatic account of the infelicity of (2) would not suffice

Challenge

- Derive incoherence of (2) while preserving non-factivity of *might*:
  a. Epistemic contradiction: $p \wedge \Diamond\neg p \models \bot$
  b. Non-factivity: $\Diamond p \not\models p$

  Classically: $p \wedge \Diamond\neg p \models \bot \Rightarrow \Diamond\neg p \models \neg p$
- Standard model-theoretic solutions use information states (Veltman, Yalcin, a.o.)

Epistemic contradictions: proof-theoretical perspective

- Suppose we can derive $\bot$ from $\Diamond\neg A \wedge A$. By classical *reductio* we would be able to derive $\neg A$ from $\Diamond\neg A$:

$$\dfrac{\dfrac{\dfrac{\Diamond\neg A \qquad [A]^1}{\Diamond\neg A \wedge A}\ (\wedge\text{-Introduction})}{\bot}\ (\text{epistemic contradiction})}{\neg A}\ (\text{classical } \textit{reductio})^1$$

- How can we prevent this derivation while preserving classicality?

First move

- Obvious culprit: classical *reductio*
- Replace *classical reductio* with weaker *epistemic reductio*:

$$(\text{classical } \textit{reductio})\ \dfrac{\begin{matrix}[A]\\ \vdots\\ \bot\end{matrix}}{\neg A} \qquad (\text{epistemic } \textit{reductio})\ \dfrac{\begin{matrix}[A]\\ \vdots\\ \bot\end{matrix}}{\Diamond\neg A}$$

- But such replacement is not sufficient

Epistemic contradictions: proof-theoretical perspective

Modals and quantification

- *Desiderata*
  1. Barcan Formulae: $\forall x \Box A \rightarrow \Box \forall x A$; $\Diamond \exists x Ax \rightarrow \exists x \Diamond Ax$ (yes)
  2. Converse BF: $\Box \forall x Ax \rightarrow \forall x \Box Ax$; $\exists x \Diamond Ax \rightarrow \Diamond \exists x Ax$ (yes)
  3. *De re–de dicto* collapse: $\forall x \Diamond Ax \rightarrow \Diamond \forall x Ax$; $\Box \exists x Ax \rightarrow \exists x \Box Ax$ (no)
  4. Converse *dr-dd* collapse: $\Diamond \forall x Ax \rightarrow \forall x \Diamond Ax$; $\exists x \Box Ax \rightarrow \Box \exists x Ax$ (yes)

Problem

- *De re–de dicto* collapse can be derived with epistemic *reductio*:

$$
\dfrac{\dfrac{\dfrac{\dfrac{[\exists x \neg Ax]^2 \qquad \dfrac{\dfrac{[\forall x \Diamond Ax]^3}{\Diamond A[y/x]}\,(\forall\text{E.}) \qquad [\neg A[y/x]]^1}{\bot}\,\text{(epistemic contradiction)}}{\bot}\,(\exists\text{E.})^1}{\Diamond \neg \exists x \neg Ax}\,\text{(epistemic } reductio)^2}{\Diamond \forall x Ax}\,\text{(duality)}}{\forall x \Diamond Ax \rightarrow \Diamond \forall x Ax}\,\text{(conditional proof)}^3
$$

$\Rightarrow$ Other classically valid principles must fail (in addition to *reductio*)

Weak assertion meets information states

- Challenge one: can we design a logical system which stays as close as possible to classical quantified modal logic but still derives the inconsistency of epistemic contradictions?
- Previous work
  - Veltman (1997) developed a state-based model theory which derived the inconsistency of epistemic contradictions without trivialising epistemic $\Diamond$;
  - Incurvati and Schlöder (2018) developed a multilateral proof theory for propositional modal logic which derives epistemic contradictions while preserving classicality.
- This paper extends the proof theory from Incurvati & Schlöder with quantifiers and provides it with a model theory which uses states (Veltman) and conceptual covers (Aloni 2001, 2005).
- Conceptual covers needed to address challenge two.
- References

  - Aloni (2005) Individual concepts in modal predicate logic. *Journal of Philosophical Logic 34*
  - Incurvati & Schlöder (2018) Weak assertion. *Philosophical Quarterly* (forthcoming)
  - Veltman (1996) Defaults in update semantics. *Journal of Philosophical Logic 25*

Reasoning in situations of partial information

> *Imagine that there is a lottery with only two tickets, a blue ticket and a red ticket. The tickets are also numbered 1 through 2, but we don't know which color goes with which number. We know that the blue ticket won.* *[Ninan 2018, page 1]*

(one) Ticket #1 is such that it might be the winning ticket.
$\exists x(x = 1 \wedge \Diamond x = w)$

(two) Ticket #2 is such that it might be the winning ticket.
$\exists x(x = 2 \wedge \Diamond x = w)$

(all) Those are all the tickets. $\forall x(x = 1 \vee x = 2)$

From these three premises we can then conclude (any): [inference 1]

(any) Any ticket might be the winning ticket.
$\forall x \Diamond x = w$

From (any), (red) seems to follow: [inference 2]

(red) The red ticket is such that it might be the winning ticket.
$\exists x(r = x \wedge \Diamond x = w)$

but (red) is false. What is wrong with this (classically valid) reasoning?

Reasoning in situations of partial information

Lottery scenario

(lot) Ticket #1 is such that it might be the winning ticket. Ticket #2 is such that it might be the winning ticket. Therefore any ticket might be the winning ticket [inference 1]. But then the red ticket is such that it might be the winning ticket [inference 2].

Informal analysis

Two salient ways to identify the tickets:

1. By number: ticket #1, ticket #2
2. By colour: the red ticket, the blue ticket

Evaluation of (any) depends on the method of identification:

(any) Any ticket might be the winning ticket.

True, if identification by number is adopted (as consequence of inference 1);
False, if identification by colour is adopted (as premise of inference 2).

Reasoning in situations of partial information

Implementation

- Identification methods formalized as *conceptual covers*, i.e.
  - sets of individuating functions from W to D such that in each world each individual is identified by at least one function (existence); in no world is an individual counted twice (uniqueness) [Aloni 2001, 2005]
- Variables range over contextually determined conceptual covers:

(any) Any$_n$ ticket might be the winning ticket.
$\forall x_n \Diamond x = w$

a. True, if $n \mapsto \{\text{ticket1, ticket2}\}$
b. False, if $n \mapsto \{\text{blue-ticket, red-ticket}\}$
c. Contradictory, if $n \mapsto \{\text{the-winning-ticket, the-losing-ticket}\}$

- Different variables can range over different covers:

(know) We don't know which$_n$ is which$_m$.
$\forall x_n \forall y_m (\Diamond x = y \wedge \Diamond x \neq y)$

- Pragmatic selection of covers governed by general principles of conversation.

Quantified Epistemic Multilateral Logic (QEML)

- The logic is based on a distinction between speech acts and contents encoded at proof-theoretical and model-theoretical level.
  - Classical logic is *unilateral* in that it models only one kind of content (on the Fregean view, content that is **asserted**)
  - *Bilateral* logics consider asserted alongside **denied** content (Smiley 1996, Rumfitt 2000)
  - Our approach is *multilateral*, considering (at least) four different attitudes: **weak/strong assertion** and **weak/strong rejection**.
- Illustration: While, classically, propositions can be either true or false, in conversation agents may display a more diversified set of attitudes towards a proposition:

(5) Is it the case that p?
  a. Yes (assenting to p)! [**strong assertion** $\mapsto +p$]
  b. No (dissenting from p)! [**strong rejection** $\mapsto -p$]
  c. Perhaps (withholding dissent from p)! [**weak assertion** $\mapsto \oplus p$]
  d. Maybe not (withholding assent to p)! [**weak rejection** $\mapsto \ominus p$]

Multilateralism: proof theory and model theory

- In a multilateral proof theory, formulae are decorated with such force markers (that means in particular that these markers do not embed).
- Model-theoretically, the clauses of the logical constants will be recursively given in terms of *update potentials*, while force markers operate globally as *tests* on information states (= sets of worlds):
  - $s \models +A$ iff $s[A] = s$ **strong assertion**
  - $s \models -A$ iff $s[A] = \emptyset$ **strong rejection**
  - $s \models \oplus A$ iff $s[A] \neq \emptyset$ or $s = \emptyset$ **weak assertion**
  - $s \models \ominus A$ iff $s[A] \neq s$ or $s = \emptyset$ **weak rejection**

Motivation for distinction force markers *vs* logical constants

1. Proof-theoretical considerations: the presence of force markers allows one to satisfy harmony constraints;
2. Ready account of difference in embeddability between *perhaps* and *might*;
3. Elegant treatment of disagreement which nicely incorporates cases of "weak disagreement".

Motivation for multilateralism: perhaps vs might

- The *might* in (6-a) has epistemic modal flavour iff (6-a) is equivalent to (6-b):

(6) a. It might be raining.
b. Perhaps it is raining.

- But *perhaps* does not embed under quantifiers, supposition and conditional antecedents, whereas *might* does.

(7) a. Every day might be your last.
b. #Every day is perhaps your last.

(8) a. Suppose it might be raining.
b. #Suppose that perhaps it is raining.

(9) a. If it might be raining, I'll take an umbrella.
b. #If perhaps it is raining, I'll take an umbrella.

- Proposal (Incurvati and Schlöder 2018):

(10) a. Perhaps p $\mapsto \oplus p$ (non-embeddable *force marker*)
b. It might be p $\mapsto +\Diamond p$ (*modifies* assertive content)

Despite this difference, QEML shows $\oplus p$ and $+\Diamond p$ to be equivalent.

Motivation for multilateralism: disagreement

Classical account

- Two agents *disagree* on a proposition *p* if they assign different truth values to *p*

Weak disagreement

- Example (11) problematic for a classical account:

(11) A: *X* or *Y* will be elected. (Grice 1991)
B: That's not so; *X* or *Y* *or* *Z* will be elected.

A and B disagree but there is no relevant *p* such that A and B would assign different truth values to *p*. B does not take '*X* or *Y* will be elected' to be false, otherwise B would have said 'Z will be elected'.

Multilateral account

- Two agents *disagree* on *p* if they display conflicting attitudes towards *p*. More precisely,
  (i) their respective attitudes towards *p* are of different polarity and
  (ii) at least one of the two has a strong attitude (assent or dissent).

Disagreement: Grice's example

- Analysis of Grice's example in QEML:

(12) A: *X* or *Y* will be elected. $\mapsto +(x \vee y)$
B: That's not so; $\mapsto \ominus(x \vee y)$
X or *Y* or *Z* will be elected. $\mapsto +(x \vee y \vee z)$

- There is then a p, namely $(x \vee y)$, such that A and B's attitudes towards p are of a different polarity, and A's attitude is strong:
  - A $\mapsto +p$ (strong positive)
  - B $\mapsto \ominus p$ (weak negative)

$\Rightarrow$ A and B predicted to disagree

- Two characteristics of QELM:
  - *Partiality*: we can have situations where neither p nor $\neg p$ is assertable:
    - $s \not\models +(x \vee y)$ $s = \{w_x, w_y, w_z\}$
    - $s \not\models +\neg(x \vee y)$
    - $s \models \ominus(x \vee y); s \models \oplus(x \vee y)$
  - *Indeterminacy*: a disjunction can be assertable without any of the disjuncts being assertable:
    - $s \models +(x \vee y \vee z)$ $s = \{w_x, w_y, w_z\}$
    - $s \not\models +x$; $s \not\models +y$; $s \not\models +z$

Disagreement: MacFarlane's example

- Other cases of disagreement (argued to be problematic for contextualist or expressivist accounts of epistemic modality) can also be easily accommodated:

(13) Sally: Joe might be in Boston. (MacFarlane 2014)
George: He can't be in Boston. I saw him in the hall 5 minutes ago.

- Analysis of MacFarlane's example in QEML:

(14) Sally: Joe might be in Boston. $\mapsto +\Diamond p$
George: He can't be in Boston. $\mapsto +\neg\Diamond p$

- QELM verifies the following equivalences:
  - $+\Diamond p \equiv \oplus p$
  - $+\neg\Diamond p \equiv -p$
- But then, Sally and George's attitudes towards p are of a different polarity, and George's attitude is strong:
  - Sally $\mapsto \oplus p$ (weak positive)
  - George $\mapsto -p$ (strong negative)

$\Rightarrow$ Sally and George predicted to disagree

Motivation for multilateralism: proof theory

- The multilateral approach allows an elegant proof theory in which Boolean connectives and modals have harmonious introduction and elimination rules (Incurvati and Schlöder 2018).
- The traditionally problematic cases of $\neg$ and $\Diamond$ are handled as embeddable counterparts of force operators and therefore can be introduced by "flip-rules" which trivially satisfy the harmony requirement.

Conjunction

$$(+\wedge\text{I.})\ \frac{+A \qquad +B}{+(A \wedge B)} \qquad (+\wedge\text{E.}_1)\ \frac{+(A \wedge B)}{+A} \qquad (+\wedge\text{E.}_2)\ \frac{+(A \wedge B)}{+B}$$

Negation

$$(\ominus\neg\text{I.})\ \frac{\oplus A}{\ominus\neg A} \qquad (\ominus\neg\text{E.})\ \frac{\ominus\neg A}{\oplus A}$$

$$(\oplus\neg\text{I.})\ \frac{\ominus A}{\oplus\neg A} \qquad (\oplus\neg\text{E.})\ \frac{\oplus\neg A}{\ominus A}$$

Modals

$$(+\Diamond\text{I.})\ \frac{\oplus A}{+\Diamond A} \qquad (+\Diamond\text{E.})\ \frac{+\Diamond A}{\oplus A}$$

$$(\oplus\Diamond\text{I.})\ \frac{\oplus A}{\oplus\Diamond A} \qquad (\oplus\Diamond\text{E.})\ \frac{\oplus\Diamond A}{\oplus A}$$

Proof theory: coordination principles

- These rules are complemented by so-called *coordination principles*, which govern the interaction of the force markers.

$$\textbf{(Rejection)}\ \frac{+A \qquad \ominus A}{\bot} \qquad \textbf{(Assertion)}\ \frac{+A}{\oplus A}$$

Smileian reductios (formalise what we called epistemic *reductio*)

$$\begin{array}{c}[+A]\\ \vdots\\ (\mathrm{SR}_1)\ \dfrac{\bot}{\ominus A}\end{array} \qquad \begin{array}{c}[\ominus A]\\ \vdots\\ (\mathrm{SR}_2)\ \dfrac{\bot}{+A}\end{array}$$

$$\textbf{(Weak Inference)}\ \frac{\begin{array}{c}[+A]\\ +\vdots\\ +B\end{array} \qquad \oplus A}{\oplus B}$$ if $(+\Diamond\mathrm{E}.)$ and $(\oplus\Diamond\mathrm{E}.)$ were not used to derive $+B$.

The restrictions placed on the subderivation in (Weak Inference) ensure that we avoid the counterintuitive derivations discussed in the introduction. The possibility to state these restrictions as the exclusion of $\Diamond$-Elimination rules is a central upshot of having "flip-rules" for epistemic modals, and hence of the multilateral approach.

Proof Theory: first order

- We use Read's (2004) strategy to formulate harmonious rules for identity and adopt the usual rules for quantification.
- The rules for *identity* and *quantification* across conceptual covers differ from those that stay within one cover (Aloni 2005).

Universal quantification

Shared covers:

$$(+\forall I.)\ \frac{+A[y_n/x_n]}{+\forall x_n A}$$ if y does not occur free in premisses or undischarged assumptions used to derive $A[y/x]$

$$(+\forall E.)\ \frac{+\forall x_n A}{+A[y_n/x_n]}$$

Mixed covers:

$$(+\forall I.)\ \frac{+A[y_m/x_n]}{+\forall x_n A}$$ if A is atomic and y_m does not occur free in premisses or undischarged assumptions used to derive $A[y_m/x_n]$

$$(+\forall E.)\ \frac{+\forall x_n A}{+A[y_m/x_n]}$$ A is atomic

Proof Theory: first order

Identity

Shared covers:

$$(+=\text{I}.^{nn}_1)\ \frac{\begin{array}{cc}[+F(x_n)] & [+F(y_n)] \\ \vdots & \vdots \\ \oplus F(y_n) & \oplus F(x_n)\end{array}}{+x_n = y_n}$$ if F does not occur in premisses and undischarged assumptions

$$(+=\text{E}.^{nn}_1)\ \frac{+x_n = y_n \qquad +F(x_n)}{\oplus F(y_n)} \qquad (+=\text{E}.^{nn}_2)\ \frac{+x_n = y_n \qquad +F(x_n)}{\oplus F(x_n)}$$

Mixed Covers:

$$(+=\text{I}.^{nm})\ \frac{\begin{array}{cc}[+F(x_n)] & [+F(y_m)] \\ \vdots & \vdots \\ +F(y_m) & +F(x_n)\end{array}}{+x_n = y_m}$$ if F does not occur in premisses and undischarged assumptions

$$(+=\text{E}.^{nm}_1)\ \frac{+x_n = y_m \qquad +F(x_n)}{+F(y_m)} \qquad (+=\text{E}.^{nm}_2)\ \frac{+x_n = y_m \qquad +F(y_m)}{+F(x_n)}$$

Model theory: models and covers

Language

$$\varphi \quad := \quad | +A \mid \oplus A \mid \ominus A \mid \bot \tag{1}$$

where A is a formula of the language of modal predicate logic with identity and indexed variables x_n where n indicates a conceptual cover:

$$A \quad := \quad Px_n, \ldots, x_n \mid x_n = x_n \mid \neg A \mid A \wedge A \mid \forall x_n A \mid \Diamond A \tag{2}$$

Models

A *model* $M = \langle W, D, I, C \rangle$ consists of a (fixed) universe W of possible worlds, a set D of individuals, a world dependent interpretation function I for predicates and an ordered set $C = \langle C_1, C_2, ... \rangle$ of conceptual covers based on (W, D).

Covers and assignments

A *conceptual cover* CC based on (W, D) is a set of functions $W \to D$ such that: $\forall w \in W : \ \forall d \in D : \ \exists! c \in CC : c(w) = d$.
An *assignment function* on (W, D, I, C) is a function g that assigns to each x_n a member of C_n.

Model theory: updates semantics for logical constants

Let M be a model, g be an assignment, and $s \subseteq W$.

$$\begin{aligned}
s[Px^1, \ldots, x^m]_g &= \{w \in s \mid \langle g(x^1)(w), \ldots, g(x^m)(w)\rangle \in I(P)(w)\} \\
s[x = y]_g &= \{w \in s \mid g(x)(w) = g(y)(w)\} \\
s[\neg A]_g &= s \setminus s[A]_g \\
s[A \wedge B]_g &= s[A]_g \cap s[B]_g \\
s[\Diamond A]_g &= s \text{ if } s[A]_g \neq \emptyset, \text{ otherwise empty} \\
s[\forall x_n A]_g &= \bigcap_{c \in C_n} s[A]_{g[x_n/c]}
\end{aligned}$$

The abbreviated operators are as expected:

$$\begin{aligned}
s[A \vee B]_g &= s[\neg(\neg A \wedge \neg B)]_g = s[A]_g \cup s[B]_g \\
s[\Box A]_g &= s[\neg\Diamond\neg A] = s \quad \text{if } s[A]_g = s, \text{ otherwise empty} \\
s[A \rightarrow B]_g &= s[\neg(A \wedge \neg B)]_g \\
s[\exists x_n A]_g &= s[\neg\forall x_n \neg A]_g = \bigcup_{c \in C_n} s[A]_{g[x_n/c]}
\end{aligned}$$

Model theory: force markers and logical consequence

Force markers as tests

Let A be a formula, M be a model, $s \subseteq W$ and g an assignment.

- $M, s, g \models +A$ iff $s[A]_g = s$.
- $M, s, g \models \oplus A$ iff $s[A]_g \neq \emptyset$ or $s = \emptyset$.
- $M, s, g \models \ominus A$ iff $s[A]_g \neq s$ or $s = \emptyset$.
- $M, s, g \models \bot$ iff $s = \emptyset$.

We can then consider $-$ an abbreviation of $+\neg$.

- $M, s, g \models -A$ iff $M, s, g \models +\neg A$ iff $s[A]_g = \emptyset$.

Logical consequence

$\Gamma \models \varphi$ iff for all M, s, g s.t. for all $\psi \in \Gamma$, $M, s, g \models \psi \Rightarrow M, s, g \models \varphi$.

Two main results

Theorem 1 (Soundness and Completeness)

$\Gamma \models \varphi$ iff $\Gamma \vdash \varphi$

Theorem 2 (First order classicality)

If $A \models B$ in classical first-order logic, then $+\sigma[A] \vdash +\sigma[B]$ in QEML.

[For a formula A of classical first-order logic, write $\sigma[A]$ for the formula of quantified modal logic that obtains from simultaneously replacing all variables x with x_i, where i is a cover index and all predicates P in A with $\sigma(P)$ where $\sigma : \text{Pred} \to \text{wff}^{QML}$ is a mapping from predicates into formulae of quantified modal logic such that whenever P is a predicate of arity n, then $\sigma(P)$ has exactly n free variables]

- Theorem 2 means that QEML sanctions as valid
  - all inferences that are valid in classical first order logic, and
  - all inferences obtained from substituting quantified modal logic formulae (with a fixed cover) into classically valid inferences.
- The converse of Theorem 2 does not hold:
  - $+Px_n \wedge \Diamond Qx_n \vdash +\Diamond(Px_n \wedge Qx_n)$ provable in QEML, but not a substitution-instance of a classically valid inference.

Applications: Epistemic contradictions and non-factivity

We derive the incoherence of epistemic contradictions while preserving the non-factivity of *might* and no *de re*–*de dicto* collapse

Proposition (Epistemic contradiction)

$+(p \wedge \Diamond\neg p) \vdash \bot$

Proof.

$$\dfrac{\dfrac{+(p \wedge \Diamond\neg p)}{+p}\ (+\wedge\text{E.}) \qquad \dfrac{\dfrac{\dfrac{\dfrac{+(p \wedge \Diamond\neg p)}{+\Diamond\neg p}\ (+\wedge\text{E.})}{\oplus\neg p}\ (+\Diamond\text{E.})}{\ominus p}\ (\oplus\neg\text{E.})}{\bot}\ (\text{Rejection})$$

□

Proposition (Non-factivity of *might*)

$+\Diamond p \not\models +p$

Proposition (No *de re*–*de dicto* collapse)

$\not\models +(\forall x_n \Diamond P x_n \rightarrow \Diamond\forall x_n P x_n)$

Proof.

Counterex.: $s = \{w_p, w_\emptyset\}$ (non-fact) & $s = \{w_{Pa}, w_{Pb}\}$ (no-collapse) □

Applications: Lottery scenario (Ninan 2018)

Imagine that there is a lottery with only two tickets, a blue ticket and a red ticket. The tickets are also numbered 1 through 2, but we don't know which color goes with which number. We know that the blue ticket won. But since we don't know whether the blue ticket is ticket #1 or ticket #2, we don't know the number of the winning ticket. [Ninan 2018, page 1]

(one) Ticket #1 is such that it might be the winning ticket.
$\exists x(x = 1 \wedge \Diamond x = w)$

(two) Ticket #2 is such that it might be the winning ticket.
$\exists x(x = 2 \wedge \Diamond x = w)$

(all) Those are all the tickets. $\forall x(x = 1 \vee x = 2)$

From these three premises we can then conclude (any): [inference 1]

(any) Any ticket might be the winning ticket.
$\forall x \Diamond x = w$

From (any), (red) seems to follow: [inference 2]

(red) The red ticket is such that it might be the winning ticket.
$\exists x(r = x \wedge \Diamond x = w)$

but (red) is false in this scenario.

Lottery scenario: analysis

- Inference 1 and inference 2 hold in QEML with a fixed cover:
  - I1 $+\exists x_n(x_n = 1 \wedge \Diamond x_n = w), +\exists x_n(x_n = 2 \wedge \Diamond x_n = w),$
    $+\forall x_n(x_n = 1 \vee x_n = 2) \vdash +\forall x_n \Diamond x_n = w$
  - I2 $+\forall x_n \Diamond x_n = w, +\exists x_n x_n = r \vdash +\exists x_n(x_n = r \wedge \Diamond x_n = w)$
- The lottery scenario can be modeled by $s = \{w_1, w_2\}$:

$$
\begin{array}{lccc}
w_1 & \mapsto & 1 & 2^{\bullet} \\
w_2 & \mapsto & 1^{\bullet} & 2
\end{array}
$$

- All examples are supported by s when interpreted under the number cover {ticket1, ticket2};
- All examples are rejected when interpreted under the colour cover {red-ticket, blue-ticket}.
- But then we have a full explanation of the apparently contrasting intuitions at the core of Ninan's puzzle:
  - Sentences (one)-(two) are assertable in the lottery scenario because naturally interpreted under the number cover;
  - The negation of (red) is assertable in the lottery scenario because naturally interpreted under the colour cover;
  - (any) assertable under the number cover (as consequence of inference 1), but not under the colour cover (as premise of inference 2).

Embedded epistemic contradictions: quantifiers

- Groenendijk et al (1996) observed that sentences like (15) are infelicitous:

(15) #Someone who is not hiding in the closet might be hiding in the closet.
$\exists x(\neg Px \wedge \Diamond Px)$

- Yalcin (2015) (crediting Declan Smithies) pointed out that sentences like (16) are instead felicitous:

(16) Not everyone who might be sick is sick.
$\neg\forall x(\Diamond Px \rightarrow Px)$

- QEML validates (17) (by first-order classicality):

(17) $+\exists x_n(\neg Px_n \wedge \Diamond Px_n) \equiv +\neg\forall x_n(\Diamond Px_n \rightarrow Px_n)$

- Moreover, $+\exists x_n(\neg Px_n \wedge \Diamond Px_n)$ is consistent.
- Thus, neither (15) nor (16) are predicted to derive contradictions, and the infelicity of (15) needs to be explained pragmatically (e.g., via properly extending notion of a P-preserving cover).

Embedded epistemic contradictions: disjunction

- Mandelkern (2018) observed that disjunctions of epistemic contradictions (*Wittgenstein disjunctions*) are infelicitous:

(18) #Might *p* and not *p* or might *q* and not *q*.

- But in QEML $+(\Diamond p \wedge \neg p) \vee (\Diamond q \wedge \neg q)$ is consistent.
- This too is a consequence of classicality: any account which treats (18) as contradictory by validating (19) must be non-classical:

(19) $(\Diamond A \wedge \neg A) \vee (\Diamond B \wedge \neg B) \models \bot$

- For instantiate A with p and B with $\neg p$ in (19). This gives us

$$(\Diamond p \wedge \neg p) \vee (\Diamond \neg p \wedge p)$$

which is truth-conditionally equivalent to $\Diamond p \vee \Diamond \neg p$. So if (19) holds, we have that

$$\Diamond p \vee \Diamond \neg p \models \bot$$

This trivialises the $\Diamond$.
- Thus, the infelicity of (18) needs to be explained pragmatically.

The reach of classicality

Closet # Someone who is not hiding in the closet might be hiding in the closet.
$\exists x(\neg Px \wedge \Diamond Px)$

Sick Not everyone who might be sick is sick.
$\neg\forall x(\Diamond Px \rightarrow Px)$

Witt-disj # Might p and not p or might q and not q.
$(\Diamond p \wedge \neg p) \vee (\Diamond q \wedge \neg q)$

Comparison

	Closet	Sick	Witt-disj
QEML	consistent*	consistent	consistent*
Dynamics	incoherent	coherent	coherent (?)
Mandelkern	contradictory	contradictory*	contradictory

* pragmatic explanation required

- QELM behaves like classical logic
- Other approaches fail to satisfy classicality
  - Dynamics: failure of double negation law (Closet $\neq$ Sick)
  - Mandelkern (2018): failure of distributivity (*Witt-disj* contradictory)

Conclusion

- A fully worked out logic for epistemic modals and quantification combining
  - Multilateral harmonious proof theory
  - Information-based model theory with conceptual covers

QEML vs classical logic vs dynamic semantics

- In QEML: partiality and non-classical inferences like
  (a) $+p \wedge \Diamond\neg p \vdash \bot$
  (b) $+Px \wedge \Diamond Qx \vdash +\Diamond(Px \wedge Qx)$
- and failure of substitutivity with mixed covers:
  (c) $+\forall x_n \Diamond Px_n \not\models +\forall y_m \Diamond Py_m$

$\Rightarrow$ QEML $\neq$ classical quantified modal logic

- But given classicality: $+A \wedge B \equiv +B \wedge A$, $+\exists xA \equiv \neg\forall x\neg A$

$\Rightarrow$ QEML $\neq$ dynamic semantics

- The reach of classicality:
  - $+(p \wedge \Diamond\neg p) \vee (q \wedge \Diamond\neg q) \not\models \bot$ Wittgenstein-disjunction
  - $+\exists x(\neg Px \wedge \Diamond Px) \not\models \bot$ Closet

Appendix: Soundness and Completeness

- The only step in the Soundness proof requiring more than conventional methods is (Weak Inference), due to its restriction. In brief, one first demonstrates that (Weak Inference) is sound for the non-modal fragment of the logic. Then, one can show that whenever there is a derivation satisfying the restrictions, there is a derivation in which all subformulae starting with a $\Diamond$ are replaced by atoms. Finally, one can show that if such a substituted derivation is truth-preserving, then so is the original proof, concluding the soundness proof (see Incurvati and Schlöder 2019 for the propositional modal logic case).
- For Completeness, note that when $+$ is understood as $\Box$, $\oplus$ as $\Diamond$ and $\ominus$ as $\Diamond\neg$, the QEML calculus given here derives the complete set of axioms for quantified modal logic with conceptual covers given in Aloni (2005) (except that Aloni's calculus also includes constant symbols). It is then easy to see that Aloni's semantics is equivalent to the update semantics presented here.

Theorem (First order classicality)

If $A \models B$ in classical first-order logic, then $+\sigma[A] \vdash +\sigma[B]$ in QEML.

[For a formula A of classical first-order logic, write $\sigma[A]$ for the formula of quantified modal logic that obtains from simultaneously replacing all variables x with x_i, where i is a cover index and all predicates P in A with $\sigma(P)$ where σ : Pred $\to$ wff^{QML} is a mapping from predicates into formulae of quantified modal logic such that whenever P is a predicate of arity n, then $\sigma(P)$ has exactly n free variables]

Proof.

One can show that the following are theorems of epistemic multilateral logic:

P1 $+((\neg A \to \neg B) \to (B \to A))$.

P2 $+((A \to (B \to C)) \to ((A \to B) \to (A \to C)))$

P3 $+(A \to (B \to A))$

Q1 $+\forall x_n.A \to A[y_n/x_n]$

Q2 $+\forall x_n.(A \to B) \to ((\forall x_n.A) \to (\forall x_n.B))$

Q3 $+A \to \forall x_n.A$ where x_n is not free in A

I1 $+x_n = x_n$

I2 $+(x_n = y_n) \to (A[x_n/z_n] \to A[y_n/z_n])$

These are Hilbert's axioms for the classical predicate logic calculus with $=$. Since QEML contains modus ponens, this means QEML derives all classically valid inferences (for a fixed cover). Since A, B,C are arbitrary, this result is closed under σ. □