Disjunction and Negation in Dynamic Semantics

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Slides: https://www.marialoni.org/resources/DynamicBSML2022-slides.pdf

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Logic and Conversation

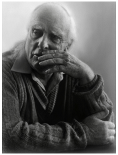
(1) ALAN: Are you going to Paul's party? Davis (SEP, 2019)
BARB: I have to work.

a. $\rightsquigarrow$ Barb has to do something [$\Rightarrow$ semantics]
b. $\rightsquigarrow$ Barb is not going to Paul's party [$\Rightarrow$ pragmatics]

Grice's paradise

Canonical divide between semantics and pragmatics

- **Pragmatic inference**: cancellable, non-embeddable, non-detachable, ... [$\Rightarrow$ derivable by conversational factors]
- **Semantic inference**: non-cancellable, embeddable, detachable, ... [$\Rightarrow$ derivable by classical logic]



H. P. Grice (1913–1988)

Beyond the canonical divide

- Gricean picture recently challenged by a class of modal inferences triggered by existential/disjunctive constructions:
  - Ignorance inference in modified numerals and epistemic indefinites;

(2) a. Aicha has *at least two* degrees $\rightsquigarrow$ speaker does't know how many [Geurts & Nouwen 2007]
b. ?I have *at least two* children.

(3) *Irgendjemand* hat angerufen. #Rat mal wer?
Irgend-someone has called Guess prt who
Someone called $\rightsquigarrow$ speaker doesn't know who [Haspelmath 1997]

  - Free choice inferences in disjunction and indefinites;

(4) a. You may go to the beach *or* to the cinema $\rightsquigarrow$ you may go to the beach *and* you may go to the cinema. [Kamp 1973]
b. $\Diamond(\alpha \vee \beta) \rightsquigarrow \Diamond\alpha \wedge \Diamond\beta$

(5) Maria muss *irgendeinen* Arzt heiraten.
Maria must irgend-one doctor marry
Mary must marry a doctor $\rightsquigarrow$ any doctor is a permissible option [Kratzer & Shimoyama 2002]

- Common core of these inferences:
  - Although derivable by conversational factors (and not by logic) they lack other defining properties of pragmatic inference [$\mapsto$ **inferences of the 3rd kind**]

Beyond Gricean paradise

		pragm. derivable	cancel lable	non-embed.	proc. cost	acqui sition
Pra gma tics	Conversational implicature B has to work $\rightsquigarrow$ B is not coming to party	$+$	$+$	$+$	high	late
Sem ant ics	Classical entailment B has to work $\rightsquigarrow$ B has to do something	$-$	$-$	$-$	low	early
3rd Kind	Epistemic indefinites *Irgendjemand* hat angerufen $\rightsquigarrow$ Speaker doesn't know who	$+$	$-$	$+$	?	?
	FC disjunction You may do A *or* B $\rightsquigarrow$ You may do A	$+$	?	?	low	early
	Scalar implicature I read *some* novels $\rightsquigarrow$ I didn't read all novels	$+$	$+$	?	high	late

N∅thing is Logical (NihiL)

- Goal of the project: a formal account of 3rd kind inferences which captures their quasi-semantic behaviour while explaining their pragmatic nature
- Strategy: develop **logics of conversation** which model next to literal meanings also pragmatic factors and the additional inferences which arise from their interaction
- Novel hypothesis: **neglect-zero** tendency as crucial pragmatic factor

> *One final remark: my specific motivation for developing this account of indicative conditionals is of course to solve a puzzle [...] But I have a broader motivation which is perhaps more important. That is to defend, by example, the claim that the concepts of pragmatics (the study of linguistic contexts) can be made as mathematically precise as any of the concepts of syntax and formal semantics; to show that one can recognize and incorporate into abstract theory the extreme context dependence which is obviously present in natural language without any sacrifice to standards of rigor [Stalnaker, 1975, Indicative Conditionals]*

Novel hypothesis: neglect-zero

- FC and ignorance inferences are
  - neither the result of conversational reasoning (as proposed in neo-gricean approaches) [$\neq$ canonical conversational implicatures]
  - nor the effect of optional applications of grammatical operators (as in the grammatical view of implicatures) [$\neq$ scalar implicatures]
- Rather they are a straightforward consequence of something else speakers do in conversation, namely,
  - **Neglect-Zero:** when interpreting a sentence people create structures representing reality (Johnson-Laird 1983) and in doing so they tend to neglect structures which (vacuously) verify the sentence by virtue of some empty configuration (*zero-models*)
- This tendency follows from the difficulty of the cognitive operation of evaluating truths with respect to empty witness sets (Nieder 2016, Bott et al, 2019)

Novel hypothesis: neglect-zero

Illustrations

(6) Every square is black.

a. Verifier: [■, ■, ■]
b. Falsifier: [■, □, ■]
c. Zero-models: [ ]; [△, △, △]; [◇, ▲, ◇]

(7) Less than three squares are black.

a. Verifier: [■, □, ■]
b. Falsifier: [■, ■, ■]
c. Zero-models: [ ]; [△, △, △]; [◇, ▲, ◇]

- Cognitive difficulty of zero-models confirmed by findings from number cognition and also explains
  - the special status of 0 among the natural numbers (Nieder, 2016)
  - existential import effects operative in the logic of Aristotle (*every square is black* $\Rightarrow$ *some square is black*) (Geurts, 2007)
  - why downward-monotonic quantifiers are more costly to process than upward-monotonic ones (Bott et al., 2019)
- Core idea: tendency to neglect zero-models, assumed to be operative in ordinary conversation, explains FC and ignorance inferences

Novel hypothesis: neglect-zero

Comparison with competing accounts

	Ignorance inference	**FC inference**	**Scalar implicature**
Neo-Gricean	reasoning	reasoning	reasoning
Grammatical view	debated	grammatical	grammatical
MA proposal	neglect-zero	neglect-zero	—

Arguments in favor of neglect-zero hypothesis

- Cognitive plausibility: differences between FC and scalar implicatures (Chemla & Bott, 2014; Tieu et al, 2016):

	processing cost	**acquisition**
FC inference	low	early
scalar implicature	high	late

  - Expected on neglect-zero hypothesis:
    - FC inference follows from the assumption that when interpreting sentences language users neglect zero-models
    - Zero-models neglected because cognitively taxing
  - Harder to explain on neo-Gricean or grammatical view
- Empirical coverage: Dual prohibition, Universal FC, Double negation FC, Wide scope FC, . . .

The paradox of free choice

- Free choice permission in natural language:

(8) You may (A or B) $\rightsquigarrow$ You may A

- But (9) not valid in standard deontic logic (von Wright 1968):

(9) $\Diamond(\alpha \vee \beta) \rightarrow \Diamond\alpha$ [Free Choice Principle]

- Plainly making the Free Choice Principle valid, for example by adding it as an axiom, would not do (Kamp 1973):

(10)
1. $\Diamond a$ [assumption]
2. $\Diamond(a \vee b)$ [from 1, by classical reasoning]
3. $\Diamond b$ [from 2, by free choice principle]

- The step leading to 2 in (10) uses the following valid principle:

(11) $\Diamond\alpha \rightarrow \Diamond(\alpha \vee \beta)$

- Natural language counterpart of (11), however, seems invalid:

(12) You may post this letter $\not\rightsquigarrow$ You may post this letter or burn it. [Ross's paradox]

$\Rightarrow$ Intuitions on natural language in direct opposition to the principles of classical logic

Reactions to paradox

- Paradox of Free Choice Permission:

(13)
1. $\Diamond a$ [assumption]
2. $\Diamond(a \vee b)$ [from 1, by addition + monotonicity]
3. $\Diamond b$ [from 2, by FC principle]

- Pragmatic solutions [$\Rightarrow$ keep logic as is]
  - FC inferences are pragmatic inferences (conversational implicatures)

  $\Rightarrow$ step leading to 3 is unjustified
- Grammatical solutions [$\Rightarrow$ keep logic as is]
  - FC inferences result from application of covert grammatical operator

  $\Rightarrow$ step leading to 3 is unjustified
- Semantic solutions [$\Rightarrow$ change the logic]
  - FC inferences are semantic entailments (e.g., Aloni 2007)

  $\Rightarrow$ step leading to 3 is justified, but step leading to 2 is no longer valid (or transitivity fails)
- Neglect-zero [$\Rightarrow$ change the logic]
  - FC inferences as neglect-zero effects (low cost pragmatics)

(14)

1.	$\Diamond a$			
2.	$\Diamond(a \vee b)$	$\neq$	$[\Diamond(a \vee b)]^+$	
3.			$\Diamond b$	

Novel hypothesis: neglect-zero

Comparison with competing accounts of FC inference

	NS FC	Dual Prohib	Universal FC	Double Neg	WS FC
Semantic	yes	no	yes	no	no
Pragmatic	yes	yes	no	?	no
Grammatical	yes	yes	yes	no	no
Neglect-zero	yes	yes	yes	yes	yes

Arguments in favor of neglect-zero hypothesis

- Empirical coverage: FC sentences give rise to complex pattern of inferences

(15)
a. $\Diamond(\alpha \vee \beta) \rightsquigarrow \Diamond\alpha \wedge \Diamond\beta$ [Narrow Scope FC]
b. $\neg\Diamond(\alpha \vee \beta) \rightsquigarrow \neg\Diamond\alpha \wedge \neg\Diamond\beta$ [Dual Prohibition]
c. $\forall x\Diamond(\alpha \vee \beta) \rightsquigarrow \forall x(\Diamond\alpha \wedge \Diamond\beta)$ [Universal FC]
d. $\neg\neg\Diamond(\alpha \vee \beta) \rightsquigarrow \Diamond\alpha \wedge \Diamond\beta$ [Double Negation FC]
e. $\Diamond\alpha \vee \Diamond\beta \rightsquigarrow \Diamond\alpha \wedge \Diamond\beta$ [Wide Scope FC]

- Captured by neglect-zero approach (Aloni 2022)
- Most other accounts need additional assumptions

Neglect-zero and dynamic semantics

- Aloni 2022: modelled neglect-zero effects in BSML (Bilateral State-based Modal Logic) using NE from team semantics
- Today: neglect-zero in dynamic semantics
  1. Bilateral Update Semantics for epistemic FC $\Leftarrow$
     - adding neglect-zero to Veltman's (1996) update semantics for MIGHT
  2. Bilateral Dynamic Semantics for anaphora and modality
     - adding neglect-zero to Groenendijk, Stokhof & Veltman's (1996) (GSV96) dynamic system for coreference & modality

References

- Aloni 2022. *Logic and Conversation: the case of free choice*. Manuscript ILLC, UvA
- Frank Veltman, 1996, Defaults in update semantics. in: *Journal of Philosophical Logic*, 25, Sections 1-3, pp. 221–231.
- Jeroen Groenendijk, Martin Stokhof and Frank Veltman, 1996. Coreference and Modality in: Shalom Lappin (ed.), *The Handbook of Contemporary Semantic Theory* Blackwell, pp. 179–213.

Neglect-zero and dynamic semantics: propositional case

- Original motivation: account for dynamic effects of epistemic modals in discourse (Veltman 1996)

(16) Maybe this is Frank Veltman's example. It isn't his example!

(17) ?This is not Frank Veltman's example! Maybe it's his example.

- But also for their FC potential (Zimmermann 2000)

(18) **Narrow scope** FC
  a. Mr. X might be in Victoria *or* in Brixton. $\rightsquigarrow$ Mr. X might be in Victoria *and* he might be in Brixton.
  b. $\Diamond(\alpha \vee \beta) \rightsquigarrow \Diamond\alpha \wedge \Diamond\beta$

(19) **Wide scope** FC
  a. Mr. X might be in Victoria or he might be in Brixton. $\rightsquigarrow$ Mr. X might be in Victoria and might be in Brixton.
  b. $\Diamond\alpha \vee \Diamond\beta \rightsquigarrow \Diamond\alpha \wedge \Diamond\beta$

- Upshot of combining dynamics and neglect-zero:
  - Interesting predictions concerning human vs mathematical reasoning (crucial is the adoption of a dynamic notion of logical consequence)

Neglect-zero and dynamic semantics: first order case

- Core motivation: account for **modal inferences** (ignorance and free choice) of epistemic indefinites

(20) *Irgendjemand* hat angerufen. #Rat mal wer?
Irgend-someone has called Guess prt who
Someone called ⇝ speaker doesn't know who [Haspelmath 1997]

(21) Maria muss *irgendeinen* Arzt heiraten.
Maria must irgend-one doctor marry
Mary must marry a doctor ⇝ any doctor is a permissible option
[Kratzer & Shimoyama 2002]

- but also their anaphoric potential:

(22) Irgendein Professor hat angerufen. Sie wollte dich sprechen.
irgend-one professor has called She wanted you speak
Some professor called. She wanted to speak with you.

(23) Wenn ein Kind irgendein Gemüse mag, dann isst er es auch roh.
When a child irgend-one vegetable likes, then eat it it also raw
When a child likes some vegetables, she also eats it raw.

- Upshot of dynamizing (q)BSML: Partee's bathroom examples

(24) Either there is no bathroom in this house or it's in a funny place.

BSML: Teams and Bilateralism

- Team semantics: formulas interpreted wrt a set of points of evaluation (a team) rather than single ones [Väänänen 2007; Yang & Väänänen 2017]

Classical vs team-based modal logic

$[M = \langle W, R, V \rangle]$

- Classical modal logic: (truth in worlds)

$$M, w \models \phi, \text{ where } w \in W$$

- Team-based modal logic:

$$M, t \models \phi, \text{ where } t \subseteq W$$

Bilateral state-based modal logic (BSML)

- Teams $\mapsto$ information states
- Assertion & rejection conditions are modeled rather than truth

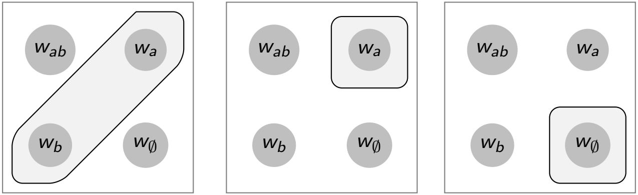
$$M, s \models \phi, \text{ “}\phi \text{ is assertable in } s\text{”}, \quad \text{with } s \subseteq W$$
$$M, s \mathrel{=\!\!|} \phi, \text{ “}\phi \text{ is rejectable in } s\text{”}, \quad \text{with } s \subseteq W$$

- Inferences relate speech acts rather than propositions and therefore might diverge from semantic entailments

Neglect-zero effects in BSML: core idea

- A state s supports a **disjunction** $\phi \vee \psi$ iff s is the union of two substates, each supporting one of the disjuncts

$$M, s \models \phi \vee \psi \text{ iff } \exists t, t' : t \cup t' = s \;\&\; M, t \models \phi \;\&\; M, t' \models \psi$$



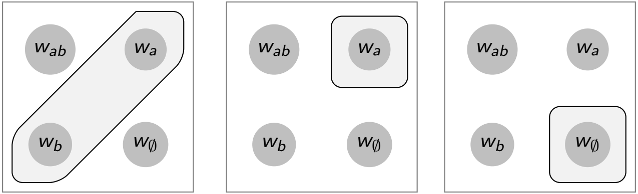
(a) Verifier (b) Zero-model (c) Falsifier

Figure: Models for $(a \vee b)$.

- $\{w_a\} \models (a \vee b)$, because we can find substates supporting each disjunct: $\{w_a\}$ itself, supports a, and $\emptyset$, vacuously supports b
- $\{w_a\}$ is then a **zero-model** for $(a \vee b)$, a model which verifies the formula by virtue of an empty witness
- BSML defines neglect-zero **enrichment** $[\;]^+$ whose core effect is to disallow such zero-models

Neglect-zero effects in BSML: core idea

- s supports an **enriched disjunction** $[\phi \vee \psi]^+$ iff s is the union of two non-empty substates, each supporting one of the disjuncts



(a) $\models \ [a \vee b]^+$ (b) $\not\models \ [a \vee b]^+$ (c) $\not\models \ [a \vee b]^+$

- An enriched disjunction $[\phi \vee \psi]^+$ requires both disjuncts to be live possibilities
- Aloni 2022 defined neglect-zero enrichment in terms of non-emptiness atom (NE) from team logic

Neglect-zero effects in BSML: implementation

- Non-emptiness atom (NE): NE requires the supporting state to be non-empty:

$$M, s \models \mathrm{NE} \quad \text{iff} \quad s \neq \emptyset$$

- Pragmatic enrichment function: Pragmatically enriched formula $[\alpha]^+$ comes with the requirement to satisfy NE distributed along each of its subformulas:

$$\begin{array}{rcl} [p]^+ & = & p \wedge \mathrm{NE} \\ [\neg\alpha]^+ & = & \neg[\alpha]^+ \wedge \mathrm{NE} \\ [\alpha \vee \beta]^+ & = & ([\alpha]^+ \vee [\beta]^+) \wedge \mathrm{NE} \\ [\alpha \wedge \beta]^+ & = & ([\alpha]^+ \wedge [\beta]^+) \wedge \mathrm{NE} \\ [\Diamond\alpha]^+ & = & \Diamond[\alpha]^+ \wedge \mathrm{NE} \end{array}$$

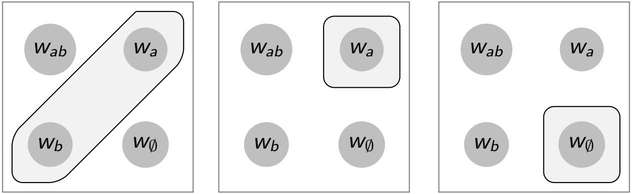
- Main result: in BSML $[\ ]^+$-enrichment has non-trivial effect only when applied to positive disjunctions:
  - $\mapsto$ we derive FC effects (for pragmatically enriched formulas);
  - $\mapsto$ pragmatic enrichment vacuous under single negation.

Update Semantics for MIGHT (Veltman 1996)

- Language: $\phi := p \mid \neg\phi \mid \phi \vee \phi \mid \phi \wedge \phi \mid \Diamond\phi$, with $p \in A$
- Models: $M = \langle W, V \rangle$, W set of worlds & V valuation function
- Information States: $s \subseteq W$, sets of possible worlds
- Updates: formulas denote functions from states to states

  (i) $s[p] = s \cap \{w \in W \mid V(p, w) = 1\}$
  (ii) $s[\phi \wedge \psi] = s[\phi] \cap s[\psi]$
  (iii) $s[\phi \vee \psi] = s[\phi] \cup s[\psi]$
  (iv) $s[\neg\phi] = s - s[\phi]$
  (v) $s[\Diamond\phi] = s$, if $s[\phi] \neq \emptyset$; $\emptyset$ otherwise

- Support: $s \models \phi$ iff $s[\phi] = s$
- Examples — for $A = \{a, b\}$



(d) $\not\models a$; $\models a \vee b$ (e) $\models a$; $\models a \vee b$ (f) $\models \neg a$; $\models \neg(a \vee b)$

Neglect-zero in dynamic semantics: core ideas

- Goal: define neglect-zero enrichment in update semantics
- Core idea: Aloni's (2022):

  $$\phi \wedge \text{NE}$$

  becomes:

  $$\phi^{\text{NE}}$$

- With NE interpreted as a **post-supposition**, a constraint that needs to be satisfied *after* the update with the relevant sentence:

  (25) $s[\phi^{\text{NE}}] = s[\phi]$, if $s[\phi] \neq \emptyset$; undefined (#) otherwise

- Compare with **presupposition** $\phi_{[\psi]}$ which must be satisfied *before* the update (in the local context):

  (26) $s[\phi_{[\psi]}] = s[\phi]$, if $s \models \psi$; undefined (#) otherwise

- And Veltman's MIGHT:

  (27) $s[\Diamond\phi] = s$, if $s[\phi] \neq \emptyset$; $\emptyset$ otherwise

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- Compare with **presupposition** $\phi_{[\psi]}$ which must be satisfied *before* the update (in the local context):

  (29) $s[\phi_{[\psi]}] = s[\phi]$, if $s \models \psi$; undefined (#) otherwise

- And Veltman's MIGHT:

  (30) $s[\Diamond\phi] = s$, if $s[\phi] \neq \emptyset$; $\emptyset$ otherwise

Neglect-zero in dynamic semantics: core ideas

- Ignorance and FC inferences easily derived for enriched **disjunctions**:

(31) $\alpha^{\mathrm{NE}} \vee \beta^{\mathrm{NE}} \models \Diamond\alpha \wedge \Diamond\beta$

(32) $\Diamond(\alpha^{\mathrm{NE}} \vee \beta^{\mathrm{NE}}) \models \Diamond\alpha \wedge \Diamond\beta$

(33) $\Diamond\alpha^{\mathrm{NE}} \vee \Diamond\beta^{\mathrm{NE}} \models \Diamond\alpha \wedge \Diamond\beta$

- But what about **negation**? Under negation (enriched) disjunction should behave classically:

(34) Mr X is not in A or B $\rightsquigarrow$ Mr X is not in A and he is not in B.

(35) Mr X cannot be in A or B $\rightsquigarrow$ Mr X cannot be in A and he cannot be in B.

- Standard dynamic negation ($s[\neg\phi] = s - s[\phi]$) gives wrong results. Formulas in (36) never supported by any state, e.g., undef in $\{w_\emptyset\}$:

(36) a. $\neg(\alpha^{\mathrm{NE}} \vee \beta^{\mathrm{NE}})$
b. $\neg\Diamond(\alpha^{\mathrm{NE}} \vee \beta^{\mathrm{NE}})$

- To fix this we will adopt a bilateral notion of negation, as in BSML, defined in terms of rejection: $s[\neg\phi] = s[\phi]^r$ & $s[\neg\phi]^r = s[\phi]$

Bilateral Update Semantics (BiUS): the propositional case

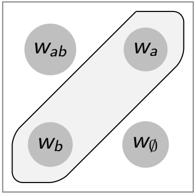
- Language

$$\phi \quad := \quad p \mid \neg\phi \mid \phi \vee \phi \mid \phi \wedge \phi \mid \Diamond\phi \mid \phi^{\mathrm{NE}}$$

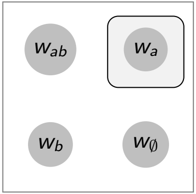
- Models: $M = \langle W, V \rangle$, W set of worlds & V valuation function
- States: $s \subseteq W$, sets of possible worlds

Examples of states

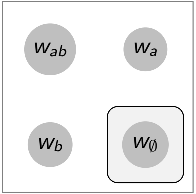
for $A = \{a, b\}$



(g) $\not\models a$; $\models |a \vee b|^+$



(h) $\models a$; $\not\models |a \vee b|^+$



(i) $\models \neg|a \vee b|^+$

Bilateral Update Semantics: the propositional case

Updates

- Formulas denote functions from states to states:

  (i) $s[p] = s \cap \{w \in W \mid V(p,w) = 1\}$
  (ii) $s[\phi \wedge \psi] = s[\phi] \cap s[\psi]$
  (iii) $s[\phi \vee \psi] = s[\phi] \cup s[\psi]$
  (iv) $s[\Diamond\phi] = s$, if $s[\phi] \neq \emptyset$; $\emptyset$ otherwise
  (v) $s[\phi^{\mathrm{NE}}] = s[\phi]$, if $s[\phi] \neq \emptyset$; undefined ($\#$) otherwise
  (vi) $s[\neg\phi] = s[\phi]^r$

- where $[\phi]^r$ is defined as follows:

  (i) $s[p]^r = s \cap \{w \in W \mid V(p,w) = 0\}$
  (ii) $s[\phi \wedge \psi]^r = s[\phi]^r \cup [\psi]^r$
  (iii) $s[\phi \vee \psi]^r = s[\phi]^r \cap [\psi]^r$
  (iv) $s[\Diamond\phi]^r = s$, if $s[\phi]^r = s$; $\emptyset$ otherwise
  (v) $s[\phi^{\mathrm{NE}}]^r = s[\phi]^r$
  (vi) $s[\neg\phi]^r = s[\phi]$

- and $s \neq \emptyset$ means s is a *state* different from $\emptyset$ (excludes $\#$)

Bilateral Update Semantics: the propositional case

Support

A state s supports ϕ, $s \models \phi$ iff $s[\phi] = s$

Logical consequence

$$\phi_1, \ldots, \phi_n \models \psi \text{ iff for all } s : s[\phi_1] \ldots [\phi_n] \text{ defined } \Rightarrow s[\phi_1] \ldots [\phi_n] \models \psi$$

Pragmatic enrichment

For $\textsc{ne}$-free α, $|\alpha|^+$ defined as follows:

$$\begin{array}{rcl}
|p|^+ & = & p^{\text{NE}} \\
|\neg\alpha|^+ & = & (\neg|\alpha|^+)^{\text{NE}} \\
|\alpha \vee \beta|^+ & = & (|\alpha|^+ \vee |\beta|^+)^{\text{NE}} \\
|\alpha \wedge \beta|^+ & = & (|\alpha|^+ \wedge |\beta|^+)^{\text{NE}} \\
|\Diamond\alpha|^+ & = & (\Diamond|\alpha|^+)^{\text{NE}}
\end{array}$$

Bilateral Update Semantics: results

Before pragmatic enrichments

- We match the predictions of Veltman (1996)
- We deal with epistemic contradictions (Yalcin, 2007) as in Veltman:
  1. Epistemic contradiction: $\neg\alpha; \Diamond\alpha \models \bot$ (but $\Diamond\alpha; \neg\alpha \not\models \bot$)
  2. Non-factivity: $\Diamond\alpha \not\models \alpha$

(37) a. Maybe this is Frank Veltman's example. It isn't his example!
b. $\Diamond\alpha; \neg\alpha$

(38) a. ?This is not Frank Veltman's example! Maybe it's his example.
b. $\neg\alpha; \Diamond\alpha$

- The ϕ^{NE}- & $\Diamond$-free fragment of BiUS is equivalent to classical logic:

$$\alpha \models_{BiUS} \beta \text{ iff } \alpha \models_{CL} \beta \qquad [\text{if } \alpha, \beta \text{ are } \phi^{\text{NE}}\text{- \& } \Diamond\text{-free}]$$

Neglect-zero effects isolated by means of NE

Bilateral Update Semantics: results

After pragmatic enrichments

- We match the predictions of Aloni 2022
- We derive ignorance and epistemic FC inferences for pragmatically enriched formulas:
  - Narrow scope FC: $|\Diamond(\alpha \vee \beta)|^+ \models \Diamond\alpha \wedge \Diamond\beta$
  - Wide scope FC: $|\Diamond\alpha \vee \Diamond\beta|^+ \models \Diamond\alpha \wedge \Diamond\beta$
  - Modal disjunction: $|\alpha \vee \beta|^+ \models \Diamond\alpha \wedge \Diamond\beta$
- while no undesirable side effects obtain with other configurations:
  - $|\neg(\alpha \vee \beta)|^+ \models \neg\alpha \wedge \neg\beta$
  - $|\neg\Diamond(\alpha \vee \beta)|^+ \models \neg\Diamond\alpha \wedge \neg\Diamond\beta$

Sketch of proofs

$|\alpha \vee \beta|^+ \models \Diamond\alpha \wedge \Diamond\beta$

$s[|\alpha \vee \beta|^+]$ is defined $\Rightarrow s[|\alpha \vee \beta|^+] = s[\alpha^{\mathrm{NE}}] \cup s[\beta^{\mathrm{NE}}]$

$\Rightarrow s[\alpha^{\mathrm{NE}}] \subseteq s[|\alpha \vee \beta|^+] \Rightarrow s[|\alpha \vee \beta|^+][\alpha] \neq \emptyset \Rightarrow s[|\alpha \vee \beta|^+] \models \Diamond\alpha$

$\Rightarrow s[\beta^{\mathrm{NE}}] \subseteq s[|\alpha \vee \beta|^+] \Rightarrow s[|\alpha \vee \beta|^+][\beta] \neq \emptyset \Rightarrow s[|\alpha \vee \beta|^+] \models \Diamond\beta$

$\Rightarrow s[|\alpha \vee \beta|^+] \models \Diamond\alpha \wedge \Diamond\beta$

Bilateral Update Semantics: results

After pragmatic enrichments

- We match the predictions of Aloni 2022
- We derive ignorance and FC inferences for pragmatically enriched formulas:
  - Narrow scope FC: $|\Diamond(\alpha \vee \beta)|^+ \models \Diamond\alpha \wedge \Diamond\beta$
  - Wide scope FC: $|\Diamond\alpha \vee \Diamond\beta|^+ \models \Diamond\alpha \wedge \Diamond\beta$
  - Modal disjunction: $|\alpha \vee \beta|^+ \models \Diamond\alpha \wedge \Diamond\beta$
- while no undesirable side effects obtain with other configurations:
  - $|\neg(\alpha \vee \beta)|^+ \models \neg\alpha \wedge \neg\beta$
  - $|\neg\Diamond(\alpha \vee \beta)|^+ \models \neg\Diamond\alpha \wedge \neg\Diamond\beta$

Sketch of proofs

$|\neg(\alpha \vee \beta)|^+ \models \neg\alpha \wedge \neg\beta$

$$
\begin{aligned}
& s[|\neg(\alpha \vee \beta)|^+] \text{ is defined } \Rightarrow \\
& s[|\neg(\alpha \vee \beta)|^+] = s[\alpha^{\mathrm{NE}} \vee \beta^{\mathrm{NE}}]^r = s[\alpha^{\mathrm{NE}}]^r \cap s[\beta^{\mathrm{NE}}]^r = s[\alpha]^r \cap s[\beta]^r \neq \emptyset \\
& \Rightarrow s[|\neg(\alpha \vee \beta)|^+] \subseteq s[\alpha]^r \Rightarrow s[|\neg(\alpha \vee \beta)|^+][\alpha]^r = s[|\neg(\alpha \vee \beta)|^+] \\
& \Rightarrow s[|\neg(\alpha \vee \beta)|^+] \subseteq s[\beta]^r \Rightarrow s[|\neg(\alpha \vee \beta)|^+][\beta]^r = s[|\neg(\alpha \vee \beta)|^+] \\
& \Rightarrow s[|\neg(\alpha \vee \beta)|^+] \models \neg\alpha \wedge \neg\beta
\end{aligned}
$$

Bilateral Update Semantics: results

Double negation

- In contrast to standard dynamic systems, we validate double negation elimination (also for non-eliminative ϕ):

$$\neg\neg\phi \equiv \phi$$

Proof: $s[\neg\neg\phi] = s[\neg\phi]^r = s[\phi]$

- Thus we validate double negation FC:

(39) $|\neg\neg\Diamond(\alpha \vee \beta)|^+ \models \Diamond\alpha \wedge \Diamond\beta$

- which gives us an account of (40) (once we add quantifiers):

(40) All-others FC (Gotzner et al. 2020)

a. Exactly one girl cannot be in Beijing or Amsterdam.
$\rightsquigarrow$ One girl cannot be in either of the two and each of the others can in Beijing and in Amsterdam

b. $\exists x(\neg\Diamond(\alpha(x) \vee \beta(x)) \wedge \forall y(y \neq x \rightarrow \neg\neg\Diamond(\alpha(y) \vee \beta(y)))) \rightsquigarrow$
$\exists x(\neg\Diamond\alpha(x) \wedge \neg\Diamond\beta(x) \wedge \forall y(y \neq x \rightarrow (\Diamond\alpha(y) \wedge \Diamond\beta(y))))$

Further applications

Double negation and bathroom examples

- When applied to dynamic systems for anaphora (e.g. GSV96) bilateral negation gives us a treatment of Partee's bathroom example:

(41) a. Either there is no bathroom in this house or it's in a funny place.
b. $\neg\exists x Px \vee Qx$

Problem in GSV96: last x not bound by $\exists x$

- Assume GSV96 account of existential quantifier, conjunction and disjunction:
  - $s[\exists x\phi] = \bigcup_{d\in D}(s[x/d][\phi])$
  - $s[\phi \wedge \psi] = s[\phi][\psi]$
  - $s[\phi \vee \psi] = \{i \in s \mid i \text{ survives in } (s[\phi] \cup s[\neg\phi][\psi])\}$
- Then no matter what rejection clause one assumes for $\exists x$, last x in (41-b) is bound by $\exists x$:

(42) $s[\neg\exists xPx] \cup s[\neg\neg\exists xPx][Qx] = s[\neg\exists xPx] \cup s[\neg\exists xPx]^r[Qx] = s[\neg\exists xPx] \cup s[\exists xPx][Qx]$

Last application: human vs mathematical reasoning

- People often reason contrary to the prescriptions of classical logic.
- Hypothesis: at least in part divergence between human and logico-mathematical reasoning is due to a neglect-zero tendency:
  - While zero-models tend to be neglected in conversation, they play a crucial role in logico-mathematical reasoning.
- Logical-mathematical reasoning captured by NE-free fragments in this framework, which are equivalent to classical logic
- According to our hypothesis there are three kinds of reasonings:
  (i) *Zero-free reasonings:* classically valid reasonings which don't involve zero-models [α, β range over NE-free formulas]

$$\alpha \models \beta \ \& \ |\alpha|^+ \models |\beta|^+$$

  (ii) *Neglect-zero fallacies:* classically invalid reasonings which are valid if we neglect zero-models, e.g., ignorance and FC inferences

$$\alpha \not\models \beta \ \& \ |\alpha|^+ \models |\beta|^+$$

  (iii) *Zero-reasonings:* classically valid reasonings which rely on zero-models

$$\alpha \models \beta \ \& \ |\alpha|^+ \not\models |\beta|^+$$

- Prediction: zero-reasonings should be harder for non-logically trained reasoners than zero-free reasonings

Disjunction introduction vs disjunctive syllogism

- Consider the following two reasoning:

(43) Disjunction introduction: A. THEREFORE A OR B.

(44) Disjunctive syllogism: A OR B; NOT A. THEREFORE B.

- Both classically valid:
  - $\alpha \models \alpha \vee \beta$
  - $\alpha \vee \beta$, $\neg\alpha \models \beta$
- But only (43) is a zero-reasoning, involves zero-models:
  - $|\alpha|^+ \not\models |\alpha \vee \beta|^+$
  - $|\alpha \vee \beta|^+$, $|\neg\alpha|^+ \models |\beta|^+$
- (43) is then predicted to be more difficult than (44)
- Old experiments seem to confirm this prediction[1]

[1] Johnson-Laird et al. Propositional reasoning by model. *Psychological Review*, 99:418–439, 1992.

The case of disjunctive syllogism

- Consider the following two versions of Disjunctive Syllogism:

(45) A OR B; NOT A. THEREFORE, B.

(46) NOT A; A OR B. THEREFORE, B.

- Both classically valid:
  - $\alpha \vee \beta; \neg\alpha \models \beta$
  - $\neg\alpha; \alpha \vee \beta \models \beta$
- But only (46) involves a zero-model, any state resulting from an update with NOT A, is a zero-model for the disjunction:
  - $|\alpha \vee \beta|^+, |\neg\alpha|^+ \models |\beta|^+$
  - $|\neg\alpha|^+, |\alpha \vee \beta|^+ \models |\beta|^+, \bot$ ($s[|\neg\alpha|^+][|\alpha \vee \beta|^+]$ is never defined!)
- (46) is then predicted to be harder than (45) (experiments needed to test this prediction)
- These predictions rely on dynamic notion of logical consequence

$$\phi_1, \ldots, \phi_n \models \psi \text{ iff for all } s : s[\phi_1]\ldots[\phi_n] \text{ defined } \Rightarrow s[\phi_1]\ldots[\phi_n] \models \psi$$

Conclusions

- Free choice: a mismatch between logic and language
- Grice's insight:
  - stronger meanings can be derived paying more "attention to the nature and importance to the conditions governing conversation"
- Standard implementation: two separate components
  - Semantics: classical logic
  - Pragmatics: Gricean reasoning

  Elegant picture, but, when applied to FC, empirically inadequate
- My proposal: FC and ignorance as neglect-zero effects
  - Aloni 2022: neglect-zero effect in BSML (a team-based modal logic)
  - Today: Neglect-zero in dynamic semantics (propositional case)
- Related (future) research:
  - Logic: proof theory (Anttila (MoL 2021), Yang, MA); bimodal perspective (Baltag, van Benthem, Bezhanishvili, MA); QBSML (MA & van Ormondt); BiUS; qBiUS
  - Language: FC cancellations (Pinton (MoL 2021), Hui (MoL 2021)); modified numerals (van Ormondt & MA); indefinites (Degano & MA); monotonicity failure under attitude verbs (Yan & MA); acquisition (children's conjunctive strengthening of disjunction); experiments.