N∅thing is Logical

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N∅thing is logical (Nihil)

- Goal of the project: a formal account of a class of natural language inferences which deviate from classical logic
- Common assumption: these deviations are not logical mistakes, but consequence of pragmatic enrichment
- Strategy: develop *logics of conversation* which model next to literal meanings also pragmatic factors and the additional inferences which arise from their interaction
- Novel hypothesis: **neglect-zero** tendency as crucial pragmatic factor
- Main conclusion: deviations from classical logic consequence of pragmatic enrichments albeit not of the canonical Gricean kind

Nihil website

`https://projects.illc.uva.nl/nihil/`

Nihil team

MA, Anttila, Brinck Knudstorp, Degano, Klochowicz & Ramotowska (+ more collaborators including Sbardolini)

Non-classical inferences

Free choice (FC)

(1) $\Diamond(\alpha \vee \beta) \rightsquigarrow \Diamond\alpha \wedge \Diamond\beta$

(2) Deontic FC inference [Kamp 1973]

a. You may go to the beach *or* to the cinema.
b. $\rightsquigarrow$ You may go to the beach *and* you may go to the cinema.

(3) Epistemic FC inference [Zimmermann 2000]

a. Mr. X might be in Victoria *or* in Brixton.
b. $\rightsquigarrow$ Mr. X might be in Victoria *and* he might be in Brixton.

Ignorance

(4) The prize is in the attic *or* in the garden $\rightsquigarrow$ speaker doesn't know where

(5) ? I have two *or* three children. [Grice 1989]

- In the standard approach, ignorance inferences are conversational implicatures
- Less consensus on FC analysed as conversational implicatures; grammatical implicatures; semantic entailments; . . .

Novel hypothesis: neglect-zero

- FC and ignorance inferences are [≠ semantic entailments]
  - Not the result of Gricean reasoning [≠ conversational implicatures]
  - Not the effect of applications of covert grammatical operators [≠ scalar implicatures]
- But rather a consequence of something else speakers do in conversation, namely,

  NEGLECT-ZERO
  when interpreting a sentence speakers create structures representing reality[1] and in doing so they systematically neglect structures which verify the sentence by virtue of an empty configuration (*zero-models*)

- Tendency to neglect zero-models follows from the difficulty of the cognitive operation of evaluating truths with respect to empty witness sets [Nieder 2016, Bott et al, 2019]

[1] Johnson-Laird (1983) *Mental Models*. Cambridge University Press.

Novel hypothesis: neglect-zero

Illustrations

(6) Every square is black.

a. Verifier: [■, ■, ■]
b. Falsifier: [■, □, ■]
c. Zero-models: [ ]; [△, △, △]; [◇, ▲, ◇]; [▲, ▲, ▲]

(7) Less than three squares are black.

a. Verifier: [■, □, ■]
b. Falsifier: [■, ■, ■]
c. Zero-models: [ ]; [△, △, △]; [◇, ▲, ◇]; [▲, ▲, ▲]; [□, □, □]

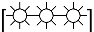
- Cognitive difficulty of zero-models confirmed by experimental findings from number cognition and has been argued to explain
  - the special status of 0 among the natural numbers [Nieder, 2016]
  - why downward-monotonic quantifiers are more costly to process than upward-monotonic ones (*less* vs *more*) [Bott et al., 2019]
  - existential import & other principles operative in Aristotelian logic (*every A is B* $\Rightarrow$ *some A is B*; *not (if not A then A))* [MA, 2023]
- Core idea: tendency to neglect zero-models, assumed to be operative in ordinary conversation, explains FC and related inferences

Novel hypothesis: neglect-zero

Illustrations

(8) It is raining.

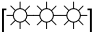
a. Verifier: []

b. Falsifier: []

c. Zero-models: none

(9) It is snowing.

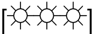
a. Verifier: []

b. Falsifier: []; []; . . .

c. Zero-models: none

(10) It is raining or snowing.

a. Verifier: [ | ]

b. Falsifier: []

c. Zero-models: []; []

- Two models in (10-c) are **zero-models** because they verify the sentence by virtue of an empty witness for one of the disjuncts
- Ignorance effects arise because such zero-models are cognitively taxing and therefore disregarded

Comparison with competing accounts

	Ignorance inference	FC inference	Scalar implicature
Neo-Gricean	reasoning	reasoning	reasoning
Grammatical view	debated	grammatical	grammatical
Nihil	neglect-zero	neglect-zero	—

Ignorance, free choice and scalar implicatures

- Scalar implicatures compatible with ignorance and free choice:

(11) Pat ate the cake or the ice-cream $\rightsquigarrow$

a. Speaker doesn't know which **(ignorance)**

b. P didn't eat both **(scalar implicature)**

(12) Pat may eat the cake or the ice-cream $\rightsquigarrow$

a. Pat may choose which $\Diamond\alpha \wedge \Diamond\beta$ **(free choice)**

b. Pat may not eat both $\neg\Diamond(\alpha \wedge \beta)$ **(scalar implicature)**

- Ignorance and free choice are incompatible

(13) Pat may eat the cake or the ice-cream, I don't know which

$\not\rightsquigarrow$ P may choose which **(free choice cancellation)**

BSML: teams and bilateralism

- Team semantics: formulas interpreted wrt a set of points of evaluation (a team) rather than single ones [Väänänen 2007; Yang & Väänänen 2017]

Classical vs team-based modal logic

$[M = \langle W, R, V \rangle]$

- Classical modal logic: (truth in worlds)

$$M, w \models \phi, \text{ where } w \in W$$

- Team-based modal logic:

$$M, t \models \phi, \text{ where } t \subseteq W$$

Bilateral state-based modal logic (BSML)

- Teams $\mapsto$ information states [Dekker93; Groenendijk$^+$96; Ciardelli$^+$19]
- Assertion & rejection conditions modeled rather than truth

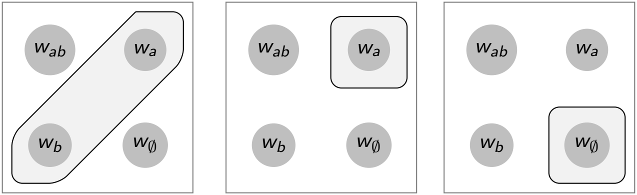
$$M, s \models \phi, \text{ "}\phi\text{ is assertable in } s\text{"}, \quad \text{with } s \subseteq W$$
$$M, s \dashv \phi, \text{ "}\phi\text{ is rejectable in } s\text{"}, \quad \text{with } s \subseteq W$$

- In BSML inferences relate speech acts rather than propositions and therefore might diverge from classical semantic entailments

Neglect-zero effects in BSML: split disjunction

- A state s supports a **disjunction** $\phi \vee \psi$ iff s is the union of two substates, each supporting one of the disjuncts

$$M, s \models \phi \vee \psi \text{ iff there are } t, t' : t \cup t' = s \ \& \ M, t \models \phi \ \& \ M, t' \models \psi$$



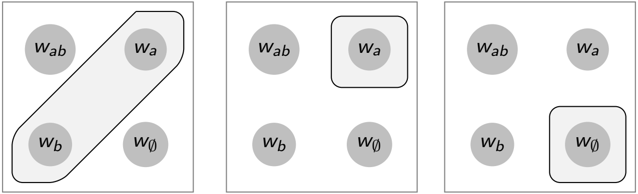
(a) No-zero verifier (b) Zero-model (c) Falsifier

Figure: Models for $(a \vee b)$.

- $\{w_a\}$ verifies $(a \vee b)$ by virtue of an empty witness for the second disjunct, $\{w_a\} = \{w_a\} \cup \emptyset$ [$\mapsto$ **zero-model**]
- Main idea: define neglect-zero enrichments, $[\ ]^+$, whose core effect is to rule out such zero-models
- Implementation: $[\ ]^+$ defined using NE ($s \models$ NE iff $s \neq \emptyset$), which models neglect-zero in the logic

Neglect-zero effects in BSML: enriched disjunction

- s supports an **enriched disjunction** $[\phi \vee \psi]^+$ iff s is the union of two non-empty substates, each supporting one of the disjuncts



(a) $\models [a \vee b]^+$ (b) $\not\models [a \vee b]^+$ (c) $\dashv [a \vee b]^+$

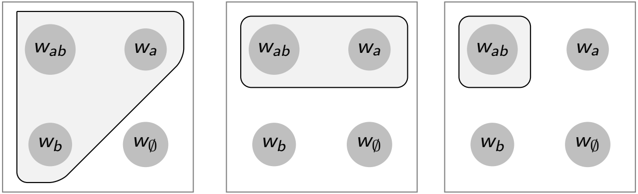
- An enriched disjunction requires both disjuncts to be live possibilities

(14) It is raining or snowing $\rightsquigarrow$ It might be raining and it might be snowing **(epistemic) possibility**

- Main result: in BSML $[\ ]^+$-enrichment has non-trivial effect only when applied to *positive* disjunctions
  - $\mapsto$ we derive FC and related effects (for pragmatically enriched formulas);
  - $\mapsto$ pragmatic enrichment vacuous under single negation.

Neglect-zero effects in BSML: possibility vs uncertainty

- More no-zero verifiers for $a \vee b$:



(d) $\models [a \vee b]^+$ (e) $\models [a \vee b]^+$ (f) $\models [a \vee b]^+$

- Two components of full ignorance ('speaker doesn't know which'):[2]

(15) It is raining or it is snowing $(\alpha \vee \beta) \rightsquigarrow$

a. Uncertainty: $\neg\Box_e\alpha \wedge \neg\Box_e\beta$

b. Possibility: $\Diamond_e\alpha \wedge \Diamond_e\beta$ (equiv $\neg\Box_e\neg\alpha \wedge \neg\Box_e\neg\beta$)

- Only possibility derived as neglect-zero effect:
  - $\{w_{ab}, w_a\} \models \Diamond_e a \wedge \Diamond_e b$, but $\not\models \neg\Box_e a$ & $\not\models \neg(a \wedge b)$
  - $\{w_{ab}, w_a\}$: a no-zero model supporting possibility but neither uncertainty nor scalar implicature [no-zero non-scalar verifier]

[2] Degano, Marty, Ramotowska, Aloni, Breheny, Romoli & Sudo. Presented at SuB & XPRAG 2023.

Two derivations of full ignorance

1. Neo-Gricean derivation [Sauerland 2004]

   (i) Uncertainty derived through quantity reasoning

   (16) $\alpha \vee \beta$ ASSERTION

   (17) $\neg\Box_e\alpha \wedge \neg\Box_e\beta$ UNCERTAINTY (from QUANTITY)

   (ii) Possibility derived from uncertainty and quality about assertion

   (18) $\Box_e(\alpha \vee \beta)$ QUALITY ABOUT ASSERTION

   (19) $\Rightarrow \Diamond_e\alpha \wedge \Diamond_e\beta$ POSSIBILITY

2. Nihil derivation

   (i) Possibility derived as neglect-zero effect

   (20) $\alpha \vee \beta$ ASSERTION

   (21) $\Diamond_e\alpha \wedge \Diamond_e\beta$ POSSIBILITY (from NEGLECT-ZERO)

   (ii) Uncertainty derived from possibility and scalar reasoning

   (22) $\neg(\alpha \wedge \beta)$ SCALAR IMPLICATURE

   (23) $\Rightarrow \neg\Box_e\alpha \wedge \neg\Box_e\beta$ UNCERTAINTY

Novel hypothesis: neglect-zero

Comparison with competing accounts

	Ignorance inference	FC inference	Scalar implicature
Neo-Gricean	reasoning	reasoning	reasoning
Grammatical view	debated	grammatical	grammatical
Nihil	neglect-zero	neglect-zero	—

- **Ignorance:** Neo-Gricean vs Nihil predictions
  - Neo-Gricean: No possibility without uncertainty
  - Nihil: Possibility derived independently from uncertainty

Argument 1 in favor of neglect-zero

- Experimental findings in agreement with Nihil predictions

  [Degano *et al*, 2023]
  - Using adapted mystery box paradigm, compared conditions in which
    - both uncertainty and possibility are false [zero-model]
    - uncertainty false but possibility true [no-zero non-scalar model]
  - Less acceptance when possibility is false (95% vs 44%)
  - Evidence that possibility can arise without uncertainty
  - A challenge for the traditional implicature approach

Novel hypothesis: neglect-zero

Comparison with competing accounts

	Ignorance inference	FC inference	Scalar implicature
Neo-Gricean	reasoning	reasoning	reasoning
Grammatical view	debated	grammatical	grammatical
Nihil	neglect-zero	neglect-zero	—

Argument 2 in favor of neglect-zero

- Cognitive plausibility: differences between FC and scalar implicatures [Chemla & Bott, 2014; Tieu et al, 2016]:

	processing cost	acquisition
FC inference	low	early
scalar implicature	high	late

- Possible explanation for neo-Gricean or grammatical view:
  - Scalar alternatives less accessible [Singh et al, 2016]
- Still low cost and early acquisition of FC
  - Hard to explain on neo-Gricean or grammatical view
  - Expected on neglect-zero hypothesis:
    - FC inference follows from the assumption that when interpreting sentences language users neglect zero-models
    - Zero-models neglected because cognitively taxing

Novel hypothesis: neglect-zero

Comparison with competing accounts of FC inference

	NS FC	Dual Prohib	Universal FC	Double Neg	WS FC
Neo-Gricean	yes	yes	no	?	no
Grammatical	yes	yes*	yes	no*	no*
Semantic	yes	no*	yes	no*	no
Neglect-zero	yes	yes	yes	yes	yes

Argument 3 in favor of neglect-zero hypothesis

- Empirical coverage: FC sentences give rise to a complex pattern of inferences

(24) a. $\Diamond(\alpha \vee \beta) \rightsquigarrow \Diamond\alpha \wedge \Diamond\beta$ [Narrow Scope FC]
b. $\neg\Diamond(\alpha \vee \beta) \rightsquigarrow \neg\Diamond\alpha \wedge \neg\Diamond\beta$ [Dual Prohibition]
c. $\forall x\Diamond(\alpha \vee \beta) \rightsquigarrow \forall x(\Diamond\alpha \wedge \Diamond\beta)$ [Universal FC]
d. $\neg\neg\Diamond(\alpha \vee \beta) \rightsquigarrow \Diamond\alpha \wedge \Diamond\beta$ [Double Negation FC]
e. $\Diamond\alpha \vee \Diamond\beta \rightsquigarrow \Diamond\alpha \wedge \Diamond\beta$ [Wide Scope FC]

- Captured by neglect-zero approach implemented in BSML[3]
- Most other approaches need additional assumptions

[3] MA (2022). Logic and conversation: the case of FC. *Sem & Pra*, 15(5).

The data

(25) Dual Prohibition [Alonso-Ovalle 2006, Marty *et al.* 2021]

a. You are not allowed to eat the cake or the ice-cream.
$\rightsquigarrow$ You are not allowed to eat either one.

b. $\neg\Diamond(\alpha \vee \beta) \rightsquigarrow \neg\Diamond\alpha \wedge \neg\Diamond\beta$

(26) Universal FC [Chemla 2009]

a. All of the boys may go to the beach or to the cinema.
$\rightsquigarrow$ All of the boys may go to the beach and all of the boys may go to the cinema.

b. $\forall x\Diamond(\alpha \vee \beta) \rightsquigarrow \forall x(\Diamond\alpha \wedge \Diamond\beta)$

(27) Double Negation FC [Gotzner *et al.* 2020]

a. Exactly one girl cannot take Spanish or Calculus.
$\rightsquigarrow$ One girl can take neither of the two and each of the others can choose between them.

b. $\exists x(\neg\Diamond(\alpha(x) \vee \beta(x)) \wedge \forall y(y \neq x \rightarrow \neg\neg\Diamond(\alpha(y) \vee \beta(y)))) \rightsquigarrow$
$\exists x(\neg\Diamond\alpha(x) \wedge \neg\Diamond\beta(x) \wedge \forall y(y \neq x \rightarrow (\Diamond\alpha(y) \wedge \Diamond\beta(y))))$

(28) Wide Scope FC [Zimmermann 2000, Hoeks *et al.* 2017]

a. Detectives may go by bus or they may go by boat.
$\rightsquigarrow$ Detectives may go by bus and may go by boat.

b. Mr. X might be in Victoria or he might be in Brixton.
$\rightsquigarrow$ Mr. X might be in Victoria and might be in Brixton.

c. $\Diamond\alpha \vee \Diamond\beta \rightsquigarrow \Diamond\alpha \wedge \Diamond\beta$

BSML

Bilateral State-Based Modal Logic (BSML)

Language

$$\phi \quad := \quad p \mid \neg\phi \mid \phi \vee \phi \mid \phi \wedge \phi \mid \Diamond\phi \mid \mathrm{NE}$$

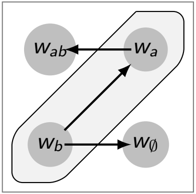
where $p \in A$.

Models and States

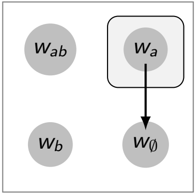
- Classical Kripke models: $M = \langle W, R, V \rangle$
- States: $s \subseteq W$, sets of worlds in a Kripke model

Examples

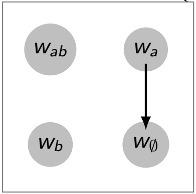
for $A = \{a, b\}$



(g) $\not\models a$; $\models \Diamond a$



(h) $\models a$; $\not\models \Diamond a$



(i) $\models a \wedge \neg a$

BSML: definitions

$[M = \langle W, R, V \rangle;\ s, t, t' \subseteq W]$

$$
\begin{array}{rll}
M, s \models p & \text{iff} & \text{for all } w \in s : V(w,p) = 1 \\
M, s \mathrel{=\!\!\mid} p & \text{iff} & \text{for all } w \in s : V(w,p) = 0 \\
M, s \models \neg\phi & \text{iff} & M, s \mathrel{=\!\!\mid} \phi \\
M, s \mathrel{=\!\!\mid} \neg\phi & \text{iff} & M, s \models \phi \\
M, s \models \phi \vee \psi & \text{iff} & \text{there are } t, t' : t \cup t' = s \ \&\ M, t \models \phi \ \&\ M, t' \models \psi \\
M, s \mathrel{=\!\!\mid} \phi \vee \psi & \text{iff} & M, s \mathrel{=\!\!\mid} \phi \ \&\ M, s \mathrel{=\!\!\mid} \psi \\
M, s \models \phi \wedge \psi & \text{iff} & M, s \models \phi \ \&\ M, s \models \psi \\
M, s \mathrel{=\!\!\mid} \phi \wedge \psi & \text{iff} & \text{there are } t, t' : t \cup t' = s \ \&\ M, t \mathrel{=\!\!\mid} \phi \ \&\ M, t' \mathrel{=\!\!\mid} \psi \\
M, s \models \Diamond\phi & \text{iff} & \text{for all } w \in s : \exists t \subseteq R[w] : t \neq \emptyset \ \&\ M, t \models \phi \\
M, s \mathrel{=\!\!\mid} \Diamond\phi & \text{iff} & \text{for all } w \in s : M, R[w] \mathrel{=\!\!\mid} \phi \\
M, s \models \mathrm{NE} & \text{iff} & s \neq \emptyset \\
M, s \mathrel{=\!\!\mid} \mathrm{NE} & \text{iff} & s = \emptyset
\end{array}
$$

where $R[w] = \{v \in W \mid wRv\}$

BSML: definitions

Box

- $\Box\phi := \neg\Diamond\neg\phi$

$$M, s \models \Box\phi \quad \text{iff} \quad \text{for all } w \in s : M, R[w] \models \phi$$

$$M, s \mathrel{=\!\!|} \Box\phi \quad \text{iff} \quad \text{for all } w \in s : \text{there is a } t \subseteq R[w] : t \neq \emptyset \ \& \ M, t \mathrel{=\!\!|} \phi$$

where $R[w] = \{v \in W \mid wRv\}$

Logical consequence

- $\phi \models \psi$ iff for all $M, s : M, s \models \phi \ \Rightarrow \ M, s \models \psi$

Proof theory

- See Anttila 2021; Anttila et al. 2022.

BSML: definitions

Pragmatic enrichment

For NE-free α, $[\alpha]^+$ defined as follows:

$$
\begin{aligned}
[p]^+ &= p \wedge \text{NE} \\
[\neg\alpha]^+ &= \neg[\alpha]^+ \wedge \text{NE} \\
[\alpha \vee \beta]^+ &= ([\alpha]^+ \vee [\beta]^+) \wedge \text{NE} \\
[\alpha \wedge \beta]^+ &= ([\alpha]^+ \wedge [\beta]^+) \wedge \text{NE} \\
[\Diamond\alpha]^+ &= \Diamond[\alpha]^+ \wedge \text{NE}
\end{aligned}
$$

State-sensitive constraints on accessibility relation

- R is indisputable in (M, s) iff $\forall w, v \in s : R[w] = R[v]$
  $\mapsto$ all worlds in s_M access exactly the same set of worlds
- R is state-based in (M, s) iff $\forall w \in s : R[w] = s$
  $\mapsto$ all and only worlds in s_M are accessible within s_M

Proposal: differences deontics vs epistemics captured by different properties of R:

- **epistemics** $\mapsto$ state-based;
- **deontics** $\mapsto$ sometimes indisputable

Neglect-zero effects in BSML: predictions

After pragmatic enrichment

- We derive both wide and narrow scope FC inferences:
  - Narrow scope FC: $[\Diamond(\alpha \vee \beta)]^+ \models \Diamond\alpha \wedge \Diamond\beta$
  - Universal FC: $[\forall x \Diamond(\alpha \vee \beta)]^+ \models \forall x(\Diamond\alpha \wedge \Diamond\beta)$
  - Double negation FC: $[\neg\neg\Diamond(\alpha \vee \beta)]^+ \models \Diamond\alpha \wedge \Diamond\beta$
  - Wide scope FC: $[\Diamond\alpha \vee \Diamond\beta]^+ \models \Diamond\alpha \wedge \Diamond\beta$ (if R is indisputable)
- while no undesirable side effects obtain with other configurations:
  - Dual prohibition: $[\neg\Diamond(\alpha \vee \beta)]^+ \models \neg\Diamond\alpha \wedge \neg\Diamond\beta$

Before pragmatic enrichment

- The NE-free fragment of BSML is equivalent to classical modal logic:

$$\alpha \models_{BSML^{\emptyset}} \beta \text{ iff } \alpha \models_{CML} \beta \qquad [\alpha, \beta \text{ are NE-free}]$$

- But we can capture the infelicity of epistemic contradictions [Yalcin, 2007] by putting team-based constraints on the accessibility relation:
  1. Epistemic contradiction: $\Diamond\alpha \wedge \neg\alpha \models \bot$ (if R is state-based)
  2. Non-factivity: $\Diamond\alpha \not\models \alpha$

Information states vs possible worlds

- Failure of bivalence in BSML

$$M, s \not\models p \ \& \ M, s \not=\!| \ p, \text{ for some info state } s$$

- Info states: less determinate than possible worlds
  - just like truthmakers, situations, possibilities, ...
- Technically:
  - Truthmakers/possibilities: points in a partially ordered set
  - Info states: sets of possible worlds, also elements of a partially ordered set, the Boolean lattice $Pow(W)$
- Thus systems using these structures are closely connected, although might diverge in motivation:
  - Truthmaker & possibility semantics: description of ontological structures in the world
  - BSML & inquisitive semantics: explaining patterns in inferential & communicative human activities
- NEXT:
  - Comparison via translations in Modal Information Logic [vBenthem19]

Comparisons via translation

- Modal Information Logic (MIL) (van Benthem, 1989, 2019):[4] common ground where related systems can be interpreted and their connections and differences can be explored
- Next: (simplified) translations into MIL of the following systems:
  - BSML
  - Truthmaker semantics (Fine)
  - Possibility semantics (Humberstone, Holliday)
  - Inquisitive semantics (Ciardelli, Groenendijk & Roelofsen)

  (cf. Gödel's (1933) translation of intuitionistic logic into modal logic)
- Focus on propositional fragments (no modalities)
  - disjunction
  - negation
- (Based on work in progress with Søren B. Knudstorp, Nick Bezhanishvili, Johan van Benthem and Alexandru Baltag)

[4] Johan van Benthem (2019) Implicit and Explicit Stances in Logic, *Journal of Philosophical Logic*.

Modal Information Logic (MIL)

Language

$$\phi \quad ::= \quad p \mid \neg\phi \mid \phi \wedge \phi \mid \phi \vee \phi \mid \langle sup \rangle \phi\psi$$

where $p \in A$.

Models and interpretation

Formulas are interpreted on triples $M = (X, \leq, V)$ where $\leq$ is a partial order

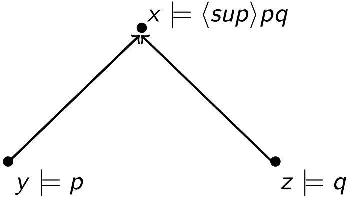
$$\begin{array}{rcl}
\mathcal{M}, x \models p & \text{iff} & x \in V(p) \\
\mathcal{M}, x \models \neg\phi & \text{iff} & \mathcal{M}, x \not\models \phi \\
\mathcal{M}, x \models \phi \wedge \psi & \text{iff} & \mathcal{M}, x \models \phi \ \text{ and } \ \mathcal{M}, x \models \psi \\
\mathcal{M}, x \models \phi \vee \psi & \text{iff} & \mathcal{M}, x \models \phi \ \text{ or } \ \mathcal{M}, x \models \psi \\
\mathcal{M}, x \models \langle sup \rangle \phi\psi & \text{iff} & \text{there are } y, z : x = sup_{\leq}(y, z) \ \& \ \mathcal{M}, y \models \phi \ \& \ \mathcal{M}, z \models \psi
\end{array}$$

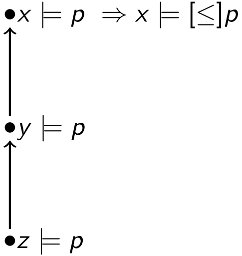
$[\leq]\phi = \neg\langle sup \rangle(\neg\varphi)\top$

$$\mathcal{M}, x \models [\leq]\phi \quad \text{iff} \quad \text{for all } y : y \leq x \ \Rightarrow \ \mathcal{M}, y \models \phi$$

Modal Information Logic (MIL)

Examples





Translations into Modal Information Logic

- BSML (non-modal NE-free fragment): $\leq$ is subset relation $\subseteq$

$$\begin{array}{rcl} & \cdots & \\ (\neg\phi)^+ & = & (\phi)^- \\ (\neg\phi)^- & = & (\phi)^+ \\ (\phi \vee \psi)^+ & = & \langle sup\rangle(\phi)^+(\psi)^+ \\ (\phi \vee \psi)^- & = & (\phi)^- \wedge (\psi)^- \\ (\phi \wedge \psi)^+ & = & (\phi)^+ \wedge (\psi)^+ \\ (\phi \wedge \psi)^- & = & \langle sup\rangle(\phi)^-(\psi)^- \\ & \cdots & \end{array}$$

- Truthmaker semantics (Fine): $\leq$ is "part of" relation

$$\begin{array}{rcl} & \cdots & \\ (\neg\phi)^+ & = & (\phi)^- \\ (\neg\phi)^- & = & (\phi)^+ \\ (\phi \vee \psi)^+ & = & (\phi)^+ \vee (\psi)^+ \\ (\phi \vee \psi)^- & = & \langle sup\rangle(\phi)^-(\psi)^- \\ (\phi \wedge \psi)^+ & = & \langle sup\rangle(\phi)^+(\psi)^+ \\ (\phi \wedge \psi)^- & = & (\phi)^- \vee (\psi)^- \\ & \cdots & \end{array}$$

Translations into Modal Information Logic

- Possibility semantics (Humberstone, Holliday)

$$\begin{array}{rcl} & \vdots & \\ tr(\neg\phi) & = & [\leq]\neg tr(\phi) \\ tr(\phi \wedge \psi) & = & tr(\phi) \wedge tr(\psi) \\ tr(\phi \vee \psi) & = & [\leq]\langle\leq\rangle(tr(\phi) \vee tr(\psi)) \\ & \vdots & \end{array}$$

- Inquisitive semantics (Groenendijk, Roelofsen and Ciardelli)

$$\begin{array}{rcl} & \vdots & \\ tr(\neg\phi) & = & [\leq]\neg tr(\phi) \\ tr(\phi \wedge \psi) & = & tr(\phi) \wedge tr(\psi) \\ tr(\phi \vee \psi) & = & tr(\phi) \vee tr(\psi) \\ & \vdots & \end{array}$$

Disjunction and Negation

- Three notions of disjunction expressible in MIL:
  - Boolean disjunction: $\phi \vee \psi$
    [classical logic, intuitionistic logic, inquisitive logic]
  - Lifted/split disjunction: $\langle sup \rangle \phi\psi$
    [BSML, dependence logic, team semantics]
  - Cofinal disjunction: $[co](\phi \vee \psi)$ (where $[co]\phi =: [\leq]\langle\leq\rangle\phi$)
    [possibility semantics, dynamic semantics]
- Three notions of negation:
  - Boolean negation: $\neg\phi$
    [classical logic, ...]
  - Bilateral negation: $(\neg\phi)^+ = (\phi)^-$ & $(\neg\phi)^- = (\phi)^+$
    [truthmaker semantics, BSML, ...]
  - Intuitionistic-like negation: $[\leq]\neg\phi$
    [possibility semantics, inquisitive semantics, intuitionistic logic]
- Some combinations:
  - Boolean disjunction + boolean negation $\mapsto$ classical logic
  - Boolean notions in other combinations can generate non-classicality:
    - Boolean disjunction + intuitionistic negation $\mapsto$ intuitionistic logic
  - Classicality also generated by non-boolean combinations:
    - Split disjunction + bilateral negation (classical fragm. BSML)

Conclusions

- Free choice and ignorance: a mismatch between logic and language
- Grice's insight:
  - stronger meanings can be derived paying more "attention to the nature and importance to the conditions governing conversation"
- Standard implementation: two separate components
  - Semantics: classical logic
  - Pragmatics: Gricean reasoning

  Elegant picture, but, when applied to FC & ignorance inferences, empirically inadequate
- My proposal: FC and ignorance as neglect-zero effects

  Literal meanings (NE-free fragment) + pragmatic factors (NE) $\Rightarrow$ FC & possibility inferences

  - Implementation in BSML (a team-based modal logic)
  - Differences but also interesting connections with related systems
  - MIL useful framework for comparisons via translations

Collaborators & related (future) research

Logic

Proof theory (Anttila, Yang, Knudstorp); expressive completeness (Anttila, Knudstorp); bimodal perspective (Knudstorp, Baltag, van Benthem, Bezhanishvili); qBSML (van Ormondt); BiUS & qBiUS (MA); typed BSML (Muskens); Aristotelian logic in $\text{qBSML}_{\rightarrow}$ (MA);...

Language

FC cancellations (Pinton, Hui); modified numerals (vOrmondt); attitude verbs (Yan); conditionals (Flachs); questions (Klochowicz); quantifiers (Klochowicz, Bott, Schlotterbeck); indefinites (Degano); homogeneity (Sbardolini); experiments (Degano, Klochowicz, Ramotowska, Bott, Schlotterbeck, Marty, Breheny, Romoli, Sudo); ...

Thank You![5]

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