# FC disjunction in state-based semantics

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# Introduction

- Free choice (FC) inferences:

(1) a. Wide scope FC: $\Diamond a \vee \Diamond b \rightsquigarrow \Diamond a \wedge \Diamond b$
    b. Narrow scope FC: $\Diamond(a \vee b) \rightsquigarrow \Diamond a \wedge \Diamond b$

- Classical examples:

(2) Deontic FC [Kamp 1973]
    a. You may go to the beach or (you may go) to the cinema.
    b. $\rightsquigarrow$ You may go to the beach and you may go to the cinema.

(3) Epistemic FC [Zimmermann 2000]
    a. Mr. X might be in Victoria or (he might be) in Brixton.
    b. $\rightsquigarrow$ Mr. X might be in Victoria and he might be in Brixton.

- Long-standing debate on the status of FC inferences:
  - FC inferences as pragmatic implicatures [Schulz, Alonso-Ovalle, Aloni, Klinedinst, Fox, Franke, Chemla, ...]
  - FC inferences as semantic entailments [Zimmermann, Geurts, Aloni, Simons, Barker, Asher & Bonevac, ...]
- Goal
  - Study notions of disjunction proposed in state-based semantics with emphasis on their potential to account for FC as a semantic or a pragmatic inference

# Why state-based semantics?

- State-based semantics (SBS): formulas interpreted wrt to a set of possible valuations rather than an individual valuation
- Particularly suitable to capture the inherent epistemic and/or alternative-inducing nature of disjunctive words in natural language

## On disjunction and uncertainty

- In languages lacking explicit *or*, disjunctive meaning expressed by adding a suffix/particle expressing uncertainty to the main verb:

(4) Johnš Billš v?aawuumšaa.
John-nom Bill-nom 3-come-pl-fut-infer
'John or Bill will come'

(5) Johnš Billš v?aawuum.
John-nom Bill-nom 3-come-pl-fut
'John and Bill will come' [Maricopa, Gil 1991, p. 102]

# Outlook

- The paradox of free choice permission
  - Pragmatic and semantic solutions
- Three notions of disjunction in state-based semantics:
  1. Classical disjunction: $\vee_1$
  2. Disjunction in team/assertability logic: $\vee_2$
  3. Disjunction in inquisitive/truthmaker semantics: $\vee_3$
- Three strategies for FC:
  - A. Pragmatic account of FC employing $\vee_1$;
  - B. Semantic account of FC employing $\vee_2$;
  - C. Semantic account of FC employing $\vee_3$.
- Focus on strategy B:
  - **System B:** A semantic account of narrow & wide scope FC using an enriched version of $\vee_2$.
- Conclusion and future work

# The paradox of free choice

- Free choice permission in natural language:

(6) You may (A or B) $\rightsquigarrow$ You may A

- But (7) not valid in standard deontic logic (von Wright 1968):

(7) $\Diamond(\alpha \vee \beta) \rightarrow \Diamond\alpha$ [Free Choice Principle]

- Plainly making the Free Choice Principle valid, for example by adding it as an axiom, would not do (Kamp 1973):

(8)
1. $\Diamond a$ [assumption]
2. $\Diamond(a \vee b)$ [from 1, by modal addition]
3. $\Diamond b$ [from 2, by free choice principle]

- The step leading to 2 in (8) uses the following valid principle:

(9) $\Diamond\alpha \rightarrow \Diamond(\alpha \vee \beta)$ [Modal Addition]

- Natural language counterpart of (9), however, seems invalid, while natural language counterpart of (7) seems to hold, in direct opposition to the principles of deontic logic:

(10) You may A $\not\rightsquigarrow$ You may (A or B)

# Reactions to paradox

- Paradox of Free Choice Permission: with extension to wide scope FC

(11)
1. $\Diamond a$ [assumption]
2. $\Diamond(a \vee b)$ / $\Diamond a \vee \Diamond b$ [from 1, by (modal) addition]
3. $\Diamond b$ [from 2, by wide/narrow scope FC principle]

- Pragmatic solutions: step leading to 3 unjustified, free choice is merely a pragmatic inference, a conversational implicature
- Semantic solutions: FC inferences as semantic entailments, step leading to 3 justified, while step leading to 2 no longer valid
- Today: pragmatic and semantic accounts of FC
  - System B (a semantic account): (modal) addition no longer valid
- Free choice: semantics or pragmatics?
  - Once we bring indefinites into the picture a purely pragmatic or a purely semantic approach to FC is untenable;
  - (Canonical) arguments for/against semantic/pragmatic approaches are inconclusive.

# Free choice: semantics or pragmatics?

## Argument in favour of pragmatic account of FC disjunction

- Free choice effects systematically disappear in negative contexts:

(12) You are not allowed to eat the cake or the ice-cream.

a. $\equiv \neg\Diamond(a \vee b) \equiv \neg\Diamond a \wedge \neg\Diamond b$

b. $\not\equiv \neg(\Diamond a \wedge \Diamond b)$

(12) never means (12-b), as would be expected if free choice effects were semantic entailments rather than pragmatic implicatures (Alonso-Ovalle 2005).

## Is this argument really conclusive?

- Our semantic system B will account for the facts in (12);
- Any pragmatic system which predicts the availability of embedded FC implicatures (like Chierchia, Fox) needs adjustments to account for these facts.
- Comparison:
  - Fox, Chierchia: non-cancellable inference $\Rightarrow$ embeddable
  - System B: FC as non-cancellable, but non-embeddable inferences

# State-based semantics

- In a state-based semantics formulas are interpreted wrt states (sets of possible worlds) rather than single possible worlds

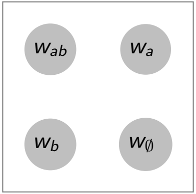
## Language

$$\phi \quad := \quad p \mid \neg\phi \mid \phi \wedge \phi \mid \phi \vee \phi \mid \Diamond\phi$$

where $p \in A$.

## States

- A state $s$ is a set of possible worlds;
- Logical space for $A = \{a, b\}$:



## Basic semantic clauses

$$s \models p \quad \text{iff} \quad \forall w \in s : w(p) = 1 \quad [p \text{ true in every world in } s]$$
$$s \models \neg\phi \quad \text{iff} \quad \forall w \in s : \{w\} \not\models \phi \quad [\phi \text{ 'false' in every world in } s]$$
$$s \models (\phi \wedge \psi) \quad \text{iff} \quad s \models \phi \ \& \ s \models \psi \quad [\text{both } \phi \ \& \ \psi \text{ supported in } s]$$

## Entailment

- $\phi \models \psi$ iff $\forall s : s \models \phi \ \Rightarrow \ s \models \psi$.

## Distributivity

- $\phi$ is distributive, if $\forall s : s \models \phi \Leftrightarrow \forall w \in s : \{w\} \models \phi$.

## Facts

- $p$, $\neg\phi$ are distributive;
- $\emptyset \models \phi$, if $\phi$ is distributive;
- So far this logic is equivalent to classical propositional logic.

# Three notions of disjunction

$s \models (\phi \vee_1 \psi)$ iff $\forall w \in s : \{w\} \models \phi$ or $\{w\} \models \psi$ (classical/dynamic semantics)

$s \models (\phi \vee_2 \psi)$ iff $\exists t, t' : t \cup t' = s$ & $t \models \phi$ & $t' \models \psi$ (team/assertability logic)

$s \models (\phi \vee_3 \psi)$ iff $s \models \phi$ or $s \models \psi$ (inquisitive/truthmaker semantics)

## Facts

1. $(\phi \vee_1 \psi) \equiv \neg(\neg\phi \wedge \neg\psi)$

If $\phi, \psi$ are distributive,

2. $(\phi \vee_1 \psi) \equiv (\phi \vee_2 \psi)$
3. $(\phi \vee_3 \psi) \models (\phi \vee_{1/2} \psi)$, but $(\phi \vee_{1/2} \psi) \not\models (\phi \vee_3 \psi)$

Counterexample

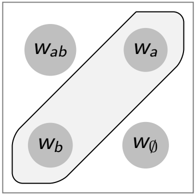



Figure: $\{w_a, w_b\} \models (a \vee_{1/2} b)$, but $\{w_a, w_b\} \not\models (a \vee_3 b)$

## Three notions of disjunction

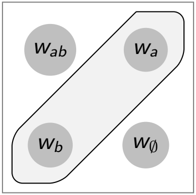
$s \models (\phi \vee_1 \psi)$ iff $\forall w \in s : \{w\} \models \phi$ or $\{w\} \models \psi$ (classical/dynamic semantics)

$s \models (\phi \vee_2 \psi)$ iff $\exists t, t' : t \cup t' = s \;\&\; t \models \phi \;\&\; t' \models \psi$ (team/assertability logic)

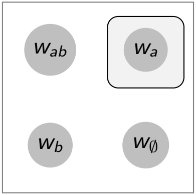
$s \models (\phi \vee_3 \psi)$ iff $s \models \phi$ or $s \models \psi$ (inquisitive/truthmaker semantics)

## Different conceptualisations for different notions of disjunction

- $\vee_{1/2}$ makes sense if $s \models \phi$ reads as
  - "agent in state $s$ has enough evidence to assert $\phi$" (assertability)
- $\vee_3$ makes sense if $s \models \phi$ reads as
  - "$\phi$ is true because of fact $s$" (truthmaker semantics)
  - "$s$ contains enough information to resolve $\phi$" (inquisitive semantics)




(a) $\not\models (a \vee_3 b)$




(b) $\models (a \vee_3 b)$

## Three notions of disjunction

$s \models (\phi \vee_1 \psi)$ iff $\forall w \in s : \{w\} \models \phi$ or $\{w\} \models \psi$ (classical/dynamic semantics)

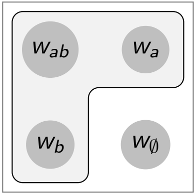
$s \models (\phi \vee_2 \psi)$ iff $\exists t, t' : t \cup t' = s \;\&\; t \models \phi \;\&\; t' \models \psi$ (team/assertability logic)

$s \models (\phi \vee_3 \psi)$ iff $s \models \phi$ or $s \models \psi$ (inquisitive/truthmaker semantics)

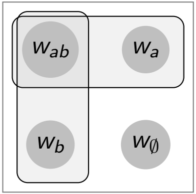
## Different semantic contents generated by different notions

Let $\phi, \psi$ be distributive and logically independent.

1. $\{s \mid s \models \phi \vee_3 \psi\}$ is inquisitive, i.e. it contains more than one maximal state, aka alternative;
2. $\{s \mid s \models \phi \vee_{1/2} \psi\}$ is not inquisitive.




(c) classical: $a \vee_{1/2} b$




(d) inquisitive: $a \vee_3 b$

# Three notions of modality

$$
\begin{array}{llll}
s \models \Diamond_1\phi & \text{iff} & \forall w \in s : R^{\rightarrow}(w) \cap info(\phi) \neq \emptyset & \text{(classical)} \\
s \models \Diamond_2\phi & \text{iff} & s \cap info(\phi) \neq \emptyset & \text{(state-based)} \\
s \models \Diamond_3\phi & \text{iff} & \forall w \in s : \forall t \in alt(\phi) : R^{\rightarrow}(w) \cap t \neq \emptyset & \text{(alternative-sensitive)}
\end{array}
$$

Auxiliary notions: $R^{\rightarrow}(w) = \{v \mid wRv\}$; $info(\phi) = \{w \mid \{w\} \models \phi\}$;
$alt(\phi) = \{s \mid s \models \phi \ \& \ \neg\exists s' : s' \models \phi \ \& \ s \subset s'\}$.

1. $\Diamond_1$ is a classical modal operator interpreted wrt a relational structure;
2. $\Diamond_2$ proposed for epistemic modals (Veltman 1981, Yalcin 2007):

   (13) #It might be raining but it is not raining.

   - Epistemic contradiction: $\Diamond_2\phi \wedge \neg\phi \models \bot$
   - Non-veridical: $\Diamond_2\phi \not\models \phi$
3. $\Diamond_3$ motivated by FC phenomena (Aloni 2007):
   - If $\phi$ is inquisitive, it generates free choice effects. Otherwise, $\Diamond_3$ behaves classically:
   - No modal contradiction: $\Diamond_3\phi \wedge \neg\phi \not\models \bot$
   - Non-veridical: $\Diamond_3\phi \not\models \phi$
   - ...

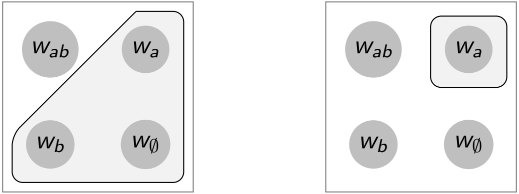
# Some facts

## Facts concerning distributivity

- State-based $\Diamond_2\phi$ is not distributive
- Classical $\Diamond_1\phi$ and alternative-sensitive $\Diamond_3\phi$ are distributive

## Facts concerning disjunction

- If $\phi, \psi$ are distributive, $(\phi \vee_1 \psi) \equiv (\phi \vee_2 \psi)$; $(\phi \vee_3 \psi) \models (\phi \vee_{1/2} \psi)$
- $(\phi \vee_2 \psi) \not\models (\phi \vee_1 \psi)$
  Counterexample: $[w_a, w_\emptyset, w_b] \models (\Diamond_2 a \vee_2 \Diamond_2 b)$, but $[w_a, w_\emptyset, w_b] \not\models (\Diamond_2 a \vee_1 \Diamond_2 b)$
- $(\phi \vee_{1/3} \psi) \not\models (\phi \vee_2 \psi)$
  Counterexample: $[w_a] \models (\Diamond_2 a \vee_{1/3} \Diamond_2 b)$, but $[w_a] \not\models (\Diamond_2 a \vee_2 \Diamond_2 b)$




(e) $\not\models \Diamond_2 a \vee_1 \Diamond_2 b$

(f) $\not\models \Diamond_2 a \vee_2 \Diamond_2 b$

## Facts about free choice

- $\vee_1$ with $\Diamond_1$ generate classical modal logic (no free choice effects)
- Assertability $\vee_2$ with state-based $\Diamond_2$ gives us wide scope FC (Hawke & Steiner-Threlkeld 2016):

$$\begin{array}{rcl} \Diamond_2 a \vee_2 \Diamond_2 b & \models & \Diamond_2 a \wedge \Diamond_2 b \\ \Diamond_2(a \vee_2 b) & \not\models & \Diamond_2 a \wedge \Diamond_2 b \end{array}$$

- Inquisitive $\vee_3$ with alternative-sensitive $\Diamond_3$ gives us narrow scope FC inference (Aloni 2007):

$$\begin{array}{rcl} \Diamond_3(a \vee_3 b) & \models & \Diamond_3 a \wedge \Diamond_3 b \\ \Diamond_3 a \vee_3 \Diamond_3 b & \not\models & \Diamond_3 a \wedge \Diamond_3 b \end{array}$$

- But problems under negation:

$$\begin{array}{rcl} \neg(\Diamond_2 a \vee_2 \Diamond_2 b) & \not\models & \neg\Diamond_2 a \wedge \neg\Diamond_2 b \\ \neg\Diamond_3(a \vee_3 b) & \not\models & \neg\Diamond_3 a \wedge \neg\Diamond_3 b \end{array}$$

## Results so far

1. Classical $\vee_1 + \Diamond_1$: no FC inference
2. Assertability $\vee_2 + \Diamond_2$: only WS epistemic FC with negation problem
3. Inquisitive $\vee_3 + \Diamond_3$: only NS FC with negation problem

## Desiderata

- An account of narrow and wide scope FC inferences;
- For epistemic and deontic modals;
- Well-behaving under negation.

## Three strategies

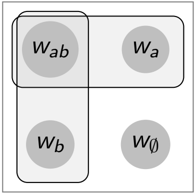
- Strategy A: Extend 1 with a pragmatic account of FC;
- Strategy B: Extend 2 with an account of NS FC;
- Strategy C: Extend 3 with an account of WS FC.

## Today focus on strategy B

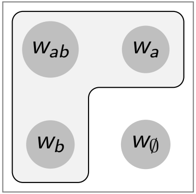
- System B: a semantic account of narrow scope and wide scope FC using enriched version of $\vee_2$

# Strategy C: FC in Inquisitive Semantics

- Or $\mapsto \vee_3$ (inquisitive)
- Modals $\mapsto \Diamond_3$ (alternative-sensitive)




(g) $a \vee_3 b$



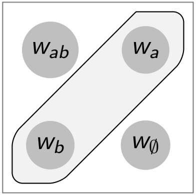

(h) $\neg\neg(a \vee_3 b)$

## Relevant results

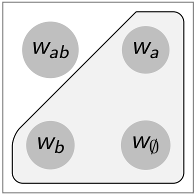
- Narrow scope FC derived as entailment: $\Diamond_3(a \vee_3 b) \models \Diamond_3 a \wedge \Diamond_3 b$
- Ways to address the negation problem:
  1. Ambiguity + strongest meaning hypothesis (e.g. Aloni 2007)
  2. Adopt a bilateral system (Roelofsen & Groenendijk, Willer, Fine)
- But no ready account of wide scope FC:
  - Epistemic WS FC can be derived by adding semantic structure (Ciardelli et al 2009). But so far no account of deontic WS FC.

# Strategy A: FC in state-based pragmatics

- Or $\mapsto \vee_1$ (classical)
- Deontic modals $\mapsto \Diamond_1$ (relational)
- Epistemic modals $\mapsto \Diamond_2$ (state-based)




(i) $opt(a \vee_1 b)$




(j) $opt(\Diamond_2(a \vee_1 b))$

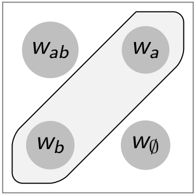
## Implicatures in a state-based semantics

- Implicatures generated via calculation of optimal states
  - Implicatures of $\phi$: what holds in any state in $opt(\phi)$ (Schulz 2005)

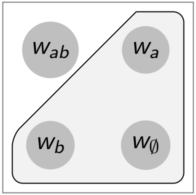
    (14) $\phi \rightsquigarrow \psi$ iff $\forall s \in opt(\phi) : s \models \psi$ and $\phi \not\models \psi$
  - Algorithms to compute $opt(\phi)$ based on Gricean principles and/or game theoretical concepts (Aloni 2007, Franke 2009, 2011)
- Incorporation of implicatures in terms of $+I$ operation (Aloni 2012)

# Strategy A: FC in state-based pragmatics

- Or $\mapsto \vee_1$ (classical)
- Deontic modals $\mapsto \Diamond_1$ (relational)
- Epistemic modals $\mapsto \Diamond_2$ (state-based)




(k) $opt(a \vee_1 b)$



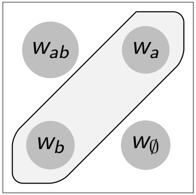

(l) $opt(\Diamond_2(a \vee_1 b))$

## Relevant results

- Narrow scope epistemic and deontic free choice derived as implicatures for both $\Diamond$ and $\Box$ (well behaving under negation);
- Only deontic FC as embeddable implicatures (Aloni & Franke 2012):
  - Prediction confirmed by experimental data (Chemla, Geurt et al) and cross-linguistic data on polarity items (Aloni & Port, Fălăuș, Crnič)
- But no account of WS FC (unless we add covert syntactic structure).

# Back to Strategy B

- $\vee_2 + \Diamond_2$ gave us wide scope FC (Hawke & Steiner-Threlkeld 2016), but:
  1. No narrow scope FC;
  2. Problems under negation;
  3. Wide scope FC only derived for epistemic modals ($\Diamond_2$ satisfies epistemic contradiction: $\Diamond_2\phi \wedge \neg\phi \models \bot$)
  4. Combination $\vee_2 + \Diamond_2$ however not really good for epistemic modals either: $\neg\Diamond_2 a \vee_2 \neg\Diamond_2 b$ compatible with $\Diamond_2 a \wedge \Diamond_2 b$



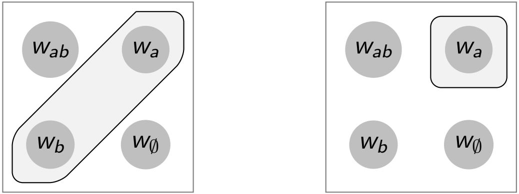

(m) $\models \neg\Diamond_2 a \vee_2 \neg\Diamond_2 b$

- System B attempts to solve all of these problems

# System B: semantic account of wide and narrow scope FC

## Disjunction

- Adopt an enriched version of $\vee_2$ (non-empty disjunction): $\Rightarrow \vee$
- A state $s$ supports a disjunction $(\phi \vee \psi)$ iff $s$ can be split into two non-empty substates, each supporting one of the disjuncts, e.g.




(n) $\models (a \vee b)$ (o) $\not\models (a \vee b)$

- $[w_a, w_b]$, $[w_{ab}]$ support $(a \vee b)$;
- but $[w_a]$ no longer supports $(a \vee b)$ [$\Leftarrow$ crucial for narrow scope FC]

# System B: semantic account of wide and narrow scope $\textsc{fc}$

## Negation facts

- To account for negation facts we adopt a bilateral system:
  - $s \vDash \phi$ interpreted as "agent in $s$ has enough evidence to assert $\phi$";
  - $s \dashv \phi$ interpreted as "agent in $s$ has enough evidence to reject $\phi$".

## Modality

- A relational (state-based) notion of modality:

$$M, s \vDash \Diamond\phi \quad \text{iff} \quad \forall_{\exists} w \in s : M, R^{\rightarrow}(w) \cap \mathit{info}(\phi) \vDash \phi$$

$$M, s \dashv \Diamond\phi \quad \text{iff} \quad \forall_{\exists} w \in s : M, R^{\rightarrow}(w) \dashv \phi$$

- Deontic vs epistemic modals:
  - Epistemics: $R$ is state-based
  - Deontics: $R$ is possibly indisputable (e.g. in performative uses)

## Outlook of results

- Narrow scope $\textsc{fc}$ derived because relevant embedded state has to support an enriched disjunction
- Wide scope $\textsc{fc}$ derived, if $R$ indisputable [state-based $\Rightarrow$ indisput.]
- Epistemic contradiction derived, if $R$ state-based [epistemics]

# System B: definitions

## Language

$$\phi \quad := \quad p \mid \neg\phi \mid \phi \wedge \phi \mid \phi \vee \phi \mid \Diamond\phi$$

where $p \in A$.

## Models

- $M = \langle W, R, S, V \rangle$, where $W$ is a set of worlds, $R$ is an accessibility relation, $S$ is a set of states (subsets of $W$), and $V$ is a world-dependent valuation function for $A$

## State-based constraints on accessibility relation

- $R$ is indisputable in $M$ iff $\forall s \in S_M : \forall w, v \in s : R^{\rightarrow}(w) = R^{\rightarrow}(v)$
  $\mapsto$ agents are fully informed about $R$
- $R$ is state-based in $M$ iff $\forall s \in S_M : \forall w \in s : R^{\rightarrow}(w) = s$
  $\mapsto$ all and only worlds in $s$ are accessible within $s$

where $R^{\rightarrow}(w) = \{v \mid wRv\}$

# System B: definitions

## Semantic clauses

$[M = \langle W, R, S, V \rangle,\ s, t, t' \subseteq W]$

$$
\begin{array}{rcl}
M, s \vdash p & \text{iff} & \forall_{\exists} w \in s : V(w, p) = 1 \\
M, s \dashv p & \text{iff} & \forall_{\exists} w \in s : V(w, p) = 0 \\
M, s \vdash \neg\phi & \text{iff} & M, s \dashv \phi \\
M, s \dashv \neg\phi & \text{iff} & M, s \vdash \phi \\
M, s \vdash \phi \wedge \psi & \text{iff} & M, s \vdash \phi \ \&\ M, s \vdash \psi \\
M, s \dashv \phi \wedge \psi & \text{iff} & \exists t, t' : t \cup t' = s \ \&\ M, t \dashv \phi \ \&\ M, t' \dashv \psi \\
M, s \vdash \phi \vee \psi & \text{iff} & \exists t, t' : t \cup t' = s \ \&\ M, t \vdash \phi \ \&\ M, t' \vdash \psi \\
M, s \dashv \phi \vee \psi & \text{iff} & M, s \dashv \phi \ \&\ M, s \dashv \psi \\
M, s \vdash \Diamond\phi & \text{iff} & \forall_{\exists} w \in s : M, R^{\rightarrow}(w) \cap \mathit{info}(\phi) \vdash \phi \\
M, s \dashv \Diamond\phi & \text{iff} & \forall_{\exists} w \in s : M, R^{\rightarrow}(w) \dashv \phi
\end{array}
$$

where $R^{\rightarrow}(w) = \{v \mid wRv\}$

# System B: definitions

## Entailment

- Strong entailment: support-entailment + rejection-entailment

$$\phi \models_S \psi \text{ iff } \forall M, s \in S_M : s \vdash \phi \;\Rightarrow\; s \vdash \psi \;\;\&\;\; s \dashv \psi \;\Rightarrow\; s \dashv \phi$$

- Weak entailment: support-entailment + dismissal-entailment

$$\phi \models \psi \text{ iff } \forall M, s \in S_M : s \vdash \phi \;\Rightarrow\; s \vdash \psi \;\;\&\;\; s \dashv \psi \;\Rightarrow\; s \nvdash \phi$$

# System B: facts about modals

- We derive narrow scope and wide scope FC as (weak) entailments:
  1. $\Diamond(a \vee b) \models \Diamond a \wedge \Diamond b$
  2. $\Diamond a \vee \Diamond b \models_S \Diamond a \wedge \Diamond b$ [if $R$ is indisputable]
- Epistemic vs deontic modals:
  - Deontic modals: $R$ typically indisputable in performative uses

    (15) We may either eat the cake or the ice-cream. [+fc, narrow]

    (16) Either we may eat the cake or the ice-cream. [–fc, wide]

    (17) You may eat the cake or you may eat the ice-cream. [+fc, wide] (Fox 2007 & Zimmermann 2000)

  - Epistemic modals: $R$ is state-based, therefore always indisputable

    (18) He might either be in London or in Paris. [+fc, narrow]

    (19) He might be in London or he might be in Paris. [+fc, wide]

    (20) ?Either he might be in London or in Paris.

    (21) #It might be raining and it is not raining.

- We derive epistemic contradiction, if $R$ is state-based:
  3. $\Diamond a \wedge \neg a \models \bot$ [if $R$ is state-based]

# System B: more facts about FC

- FC effects are more fine-grained than in inquisitive semantics:
  1. $\Diamond(a \vee (a \wedge b)) \models \Diamond a \wedge \Diamond(a \wedge b)$
  2. $\Diamond a \vee \Diamond(a \wedge b) \models_S \Diamond a \wedge \Diamond(a \wedge b)$ [if $R$ is indisputable]
- FC effects also for plain disjunction and $\Box$:
  3. $(a \vee b) \models \Diamond a \wedge \Diamond b$ [if $R$ is state-based]
  4. $\Box(a \vee b) \models \Diamond a \wedge \Diamond b$ $(\Box \equiv \neg\Diamond\neg)$
- FC effects disappear under negation:
  5. $\neg\Diamond(a \vee b) \models \neg\Diamond a \wedge \neg\Diamond b$
  6. $\neg(\Diamond a \vee \Diamond b) \models \neg\Diamond a \wedge \neg\Diamond b$
  7. $\neg(a \vee b) \models_S \neg a \wedge \neg b$
- But, behaviour under negation is postulated rather than predicted:
  - Allowing to pre-encode what should happen under negation, bilateral systems are more descriptive than explanatory (Cardelli)

# System B: some logical properties

- Double negation law:
  - $\phi \equiv \neg\neg\phi$
- De Morgan laws:
  - $\neg(\phi \vee \psi) \equiv \neg\phi \wedge \neg\psi$
  - $\neg(\phi \wedge \psi) \equiv \neg\phi \vee \neg\psi$
- Logic is highly non-standard, e.g. we lose atomic addition:
  - $a \not\models (a \vee b)$

## For comparison

- Hawke & Steiner-Threlkeld 2016:
  - $a \models_{HST} (a \vee b)$
  - $\Diamond a \not\models_{HST} (\Diamond a \vee \Diamond b)$
- Aloni 2007:
  - $\phi \models_{A} (\phi \vee \psi)$
  - $\Diamond a \not\models_{A} \Diamond(a \vee b)$

## Summary

- Three notions of disjunction in state-based semantics:
  1. Classical: $\vee_1$
  2. Team/assertability logic: $\vee_2$
  3. Inquisitive/truthmaker semantics: $\vee_3$
- System B: semantic account of FC using an enriched version of $\vee_2$:
  - Narrow scope FC as entailments (well-behaving under negation)
  - Wide scope FC as entailments (dependent on accessibility relation)
  - FC effects also for plain disjunction and under $\Box$
- Other strategies lacked a ready account of wide scope FC:
  - Strategy A: classical $\vee_1$ + state-based pragmatics
    - narrow scope FC as implicatures (both $\Diamond$ and $\Box$)
    - only deontic FC as embeddable implicature
    - no account of wide scope FC (unless we add syntactic structure)
  - Strategy C: inquisitive $\vee_3$ + alternative-sensitive $\Diamond_3$
    - narrow scope FC as entailments
    - no account of wide scope FC (unless we add semantic structure)

## Future work

- Experimentally test predictions [January 2016, Alexandre Cremers]
- Logical properties of System B;
- ...

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