

FC disjunction in state-based semantics

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Introduction

▶ Free choice (FC) inferences:

- (1) a. Wide scope FC: $\diamond a \vee \diamond b \rightsquigarrow \diamond a \wedge \diamond b$
b. Narrow scope FC: $\diamond(a \vee b) \rightsquigarrow \diamond a \wedge \diamond b$

▶ Classical examples:

- (2) Deontic FC [Kamp 1973]
a. You may go to the beach or (you may go) to the cinema.
b. \rightsquigarrow You may go to the beach and you may go to the cinema.
- (3) Epistemic FC [Zimmermann 2000]
a. Mr. X might be in Victoria or (he might be) in Brixton.
b. \rightsquigarrow Mr. X might be in Victoria and he might be in Brixton.

▶ Long-standing debate on the status of FC inferences:

- ▶ FC inferences as pragmatic implicatures
[Schulz, Alonso-Ovalle, Aloni, Klinedinst, Fox, Franke, Chemla, ...]
- ▶ FC inferences as semantic entailments
[Zimmermann, Geurts, Aloni, Simons, Barker, Asher & Bonevac, ...]

▶ GOAL

- ▶ Study notions of **disjunction** proposed in **state-based semantics** with emphasis on their potential to account for FC as a semantic or a pragmatic inference

Why state-based semantics?

- ▶ **State-based semantics (SBS)**: formulas interpreted wrt to a set of possible valuations rather than an individual valuation
- ▶ Particularly suitable to capture the inherent epistemic and/or alternative-inducing nature of disjunctive words in natural language

On disjunction and uncertainty

- ▶ In languages lacking explicit *or*, disjunctive meaning expressed by adding a suffix/particle expressing uncertainty to the main verb:

(4) Johnš Billš v?aawuumšaa.
John-nom Bill-nom 3-come-pl-fut-infer
'John or Bill will come'

(5) Johnš Billš v?aawuum.
John-nom Bill-nom 3-come-pl-fut
'John and Bill will come'

[Maricopa, Gil 1991, p. 102]

Outlook

- ▶ The paradox of free choice permission
 - ▶ Pragmatic and semantic solutions
- ▶ Three notions of disjunction in state-based semantics:
 1. Classical disjunction: \vee_1
 2. Disjunction in team/assertability logic: \vee_2
 3. Disjunction in inquisitive/truthmaker semantics: \vee_3
- ▶ Three strategies for FC:
 - A. Pragmatic account of FC employing \vee_1 ;
 - B. Semantic account of FC employing \vee_2 ;
 - C. Semantic account of FC employing \vee_3 .
- ▶ Focus on strategy B:
 - ▶ **System B**: A semantic account of **narrow & wide scope FC** using an enriched version of \vee_2 .
- ▶ Conclusion and future work

The paradox of free choice

- ▶ Free choice permission in natural language:

(6) You may (A or B) \rightsquigarrow You may A

- ▶ But (7) not valid in standard deontic logic (von Wright 1968):

(7) $\diamond(\alpha \vee \beta) \rightarrow \diamond\alpha$ [Free Choice Principle]

- ▶ Plainly making the Free Choice Principle valid, for example by adding it as an axiom, would not do (Kamp 1973):

(8) 1. $\diamond a$ [assumption]
2. $\diamond(a \vee b)$ [from 1, by modal addition]
3. $\diamond b$ [from 2, by free choice principle]

- ▶ The step leading to 2 in (8) uses the following valid principle:

(9) $\diamond\alpha \rightarrow \diamond(\alpha \vee \beta)$ [Modal Addition]

- ▶ Natural language counterpart of (9), however, seems invalid, while natural language counterpart of (7) seems to hold, in direct opposition to the principles of deontic logic:

(10) You may A $\not\rightsquigarrow$ You may (A or B)

Reactions to paradox

- ▶ Paradox of Free Choice Permission: with extension to wide scope FC

$$(11) \quad \begin{array}{ll} 1. & \diamond a \quad \text{[assumption]} \\ 2. & \diamond(a \vee b) / \diamond a \vee \diamond b \quad \text{[from 1, by (modal) addition]} \\ 3. & \diamond b \quad \text{[from 2, by wide/narrow scope FC principle]} \end{array}$$

- ▶ **Pragmatic solutions:** step leading to 3 unjustified, free choice is merely a pragmatic inference, a conversational implicature
- ▶ **Semantic solutions:** FC inferences as semantic entailments, step leading to 3 justified, while step leading to 2 no longer valid
- ▶ **Today:** pragmatic and semantic accounts of FC
 - ▶ System B (a semantic account): (modal) addition no longer valid
- ▶ Free choice: semantics or pragmatics?
 - ▶ Once we bring **indefinites** into the picture a purely pragmatic or a purely semantic approach to FC is untenable;
 - ▶ (Canonical) arguments for/against semantic/pragmatic approaches are inconclusive.

Free choice: semantics or pragmatics?

Argument in favour of pragmatic account of FC disjunction

- ▶ Free choice effects systematically disappear in negative contexts:

(12) You are not allowed to eat the cake or the ice-cream.

- a. $\equiv \neg\Diamond(a \vee b) \equiv \neg\Diamond a \wedge \neg\Diamond b$
- b. $\neq \neg(\Diamond a \wedge \Diamond b)$

(12) never means (12-b), as would be expected if free choice effects were semantic entailments rather than pragmatic implicatures (Alonso-Ovalle 2005).

Is this argument really conclusive?

- ▶ Our semantic system B will account for the facts in (12);
- ▶ Any pragmatic system which predicts the availability of embedded FC implicatures (like Chierchia, Fox) needs adjustments to account for these facts.
- ▶ Comparison:
 - ▶ Fox, Chierchia: non-cancellable inference \Rightarrow embeddable
 - ▶ System B: FC as non-cancellable, but non-embeddable inferences

State-based semantics

- ▶ In a state-based semantics formulas are interpreted wrt states (sets of possible worlds) rather than single possible worlds

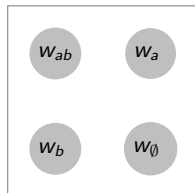
Language

$$\phi := p \mid \neg\phi \mid \phi \wedge \phi \mid \phi \vee \phi \mid \diamond\phi$$

where $p \in A$.

States

- ▶ A state s is a set of possible worlds;
- ▶ Logical space for $A = \{a, b\}$:



Basic semantic clauses

- $s \models p$ iff $\forall w \in s : w(p) = 1$ [p true in every world in s]
 $s \models \neg\phi$ iff $\forall w \in s : \{w\} \not\models \phi$ [ϕ 'false' in every world in s]
 $s \models (\phi \wedge \psi)$ iff $s \models \phi$ & $s \models \psi$ [both ϕ & ψ supported in s]

Entailment

- ▶ $\phi \models \psi$ iff $\forall s : s \models \phi \Rightarrow s \models \psi$.

Distributivity

- ▶ ϕ is **distributive**, if $\forall s : s \models \phi \Leftrightarrow \forall w \in s : \{w\} \models \phi$.

Facts

- ▶ $p, \neg\phi$ are distributive;
- ▶ $\emptyset \models \phi$, if ϕ is distributive;
- ▶ So far this logic is equivalent to classical propositional logic.

Three notions of disjunction

$s \models (\phi \vee_1 \psi)$ iff $\forall w \in s : \{w\} \models \phi$ or $\{w\} \models \psi$ (classical/dynamic semantics)

$s \models (\phi \vee_2 \psi)$ iff $\exists t, t' : t \cup t' = s$ & $t \models \phi$ & $t' \models \psi$ (team/assertability logic)

$s \models (\phi \vee_3 \psi)$ iff $s \models \phi$ or $s \models \psi$ (inquisitive/truthmaker semantics)

Facts

1. $(\phi \vee_1 \psi) \equiv \neg(\neg\phi \wedge \neg\psi)$

If ϕ, ψ are distributive,

2. $(\phi \vee_1 \psi) \equiv (\phi \vee_2 \psi)$

3. $(\phi \vee_3 \psi) \models (\phi \vee_{1/2} \psi)$, but $(\phi \vee_{1/2} \psi) \not\models (\phi \vee_3 \psi)$

Counterexample

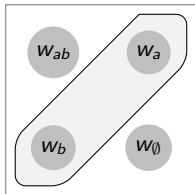


Figure: $\{w_a, w_b\} \models (a \vee_{1/2} b)$, but $\{w_a, w_b\} \not\models (a \vee_3 b)$

Three notions of disjunction

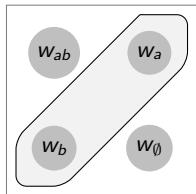
$s \models (\phi \vee_1 \psi)$ iff $\forall w \in s : \{w\} \models \phi$ or $\{w\} \models \psi$ (classical/dynamic semantics)

$s \models (\phi \vee_2 \psi)$ iff $\exists t, t' : t \cup t' = s$ & $t \models \phi$ & $t' \models \psi$ (team/assertability logic)

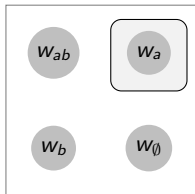
$s \models (\phi \vee_3 \psi)$ iff $s \models \phi$ or $s \models \psi$ (inquisitive/truthmaker semantics)

Different conceptualisations for different notions of disjunction

- ▶ $\vee_{1/2}$ makes sense if $s \models \phi$ reads as
 - ▶ “agent in state s has enough evidence to assert ϕ ” (assertability)
- ▶ \vee_3 makes sense if $s \models \phi$ reads as
 - ▶ “ ϕ is true because of fact s ” (truthmaker semantics)
 - ▶ “ s contains enough information to resolve ϕ ” (inquisitive semantics)



(a) $\not\models (a \vee_3 b)$



(b) $\models (a \vee_3 b)$

Three notions of disjunction

$s \models (\phi \vee_1 \psi)$ iff $\forall w \in s : \{w\} \models \phi$ or $\{w\} \models \psi$ (classical/dynamic semantics)

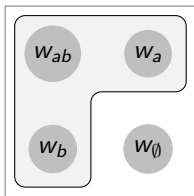
$s \models (\phi \vee_2 \psi)$ iff $\exists t, t' : t \cup t' = s$ & $t \models \phi$ & $t' \models \psi$ (team/assertability logic)

$s \models (\phi \vee_3 \psi)$ iff $s \models \phi$ or $s \models \psi$ (inquisitive/truthmaker semantics)

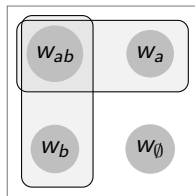
Different semantic contents generated by different notions

Let ϕ, ψ be distributive and logically independent.

1. $\{s \mid s \models \phi \vee_3 \psi\}$ is **inquisitive**, i.e. it contains more than one maximal state, aka **alternative**;
2. $\{s \mid s \models \phi \vee_{1/2} \psi\}$ is not inquisitive.



(c) classical: $a \vee_{1/2} b$



(d) inquisitive: $a \vee_3 b$

Three notions of modality

$s \models \diamond_1 \phi$ iff $\forall w \in s : R^\rightarrow(w) \cap \text{info}(\phi) \neq \emptyset$ (classical)

$s \models \diamond_2 \phi$ iff $s \cap \text{info}(\phi) \neq \emptyset$ (state-based)

$s \models \diamond_3 \phi$ iff $\forall w \in s : \forall t \in \text{alt}(\phi) : R^\rightarrow(w) \cap t \neq \emptyset$ (alternative-sensitive)

Auxiliary notions: $R^\rightarrow(w) = \{v \mid wRv\}$; $\text{info}(\phi) = \{w \mid \{w\} \models \phi\}$;

$\text{alt}(\phi) = \{s \mid s \models \phi \ \& \ \neg \exists s' : s' \models \phi \ \& \ s \subset s'\}$.

1. \diamond_1 is a classical modal operator interpreted wrt a relational structure;
2. \diamond_2 proposed for **epistemic** modals (Veltman 1981, Yalcin 2007):

(13) #It might be raining but it is not raining.

- ▶ Epistemic contradiction: $\diamond_2 \phi \wedge \neg \phi \models \perp$
- ▶ Non-veridical: $\diamond_2 \phi \not\models \phi$

3. \diamond_3 motivated by FC phenomena (Aloni 2007):

- ▶ If ϕ is inquisitive, it generates free choice effects. Otherwise, \diamond_3 behaves classically:
- ▶ No modal contradiction: $\diamond_3 \phi \wedge \neg \phi \not\models \perp$
- ▶ Non-veridical: $\diamond_3 \phi \not\models \phi$
- ▶ ...

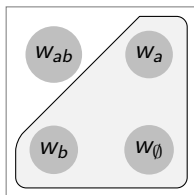
Some facts

Facts concerning distributivity

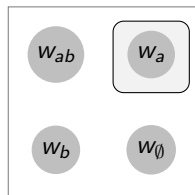
- ▶ State-based $\diamond_2\phi$ is not distributive
- ▶ Classical $\diamond_1\phi$ and alternative-sensitive $\diamond_3\phi$ are distributive

Facts concerning disjunction

- ▶ If ϕ, ψ are distributive, $(\phi \vee_1 \psi) \equiv (\phi \vee_2 \psi)$; $(\phi \vee_3 \psi) \models (\phi \vee_{1/2} \psi)$
- ▶ $(\phi \vee_2 \psi) \not\models (\phi \vee_1 \psi)$
Counterexample: $[w_a, w_\emptyset, w_b] \models (\diamond_2 a \vee_2 \diamond_2 b)$, but $[w_a, w_\emptyset, w_b] \not\models (\diamond_2 a \vee_1 \diamond_2 b)$
- ▶ $(\phi \vee_{1/3} \psi) \not\models (\phi \vee_2 \psi)$
Counterexample: $[w_a] \models (\diamond_2 a \vee_{1/3} \diamond_2 b)$, but $[w_a] \not\models (\diamond_2 a \vee_2 \diamond_2 b)$



(e) $\not\models \diamond_2 a \vee_1 \diamond_2 b$



(f) $\not\models \diamond_2 a \vee_2 \diamond_2 b$

Facts about free choice

- ▶ \vee_1 with \diamond_1 generate classical modal logic (no free choice effects)
- ▶ Assertability \vee_2 with state-based \diamond_2 gives us **wide scope** FC (Hawke & Steiner-Threlkeld 2016):

$$\begin{aligned}\diamond_2 a \vee_2 \diamond_2 b &\models \diamond_2 a \wedge \diamond_2 b \\ \diamond_2(a \vee_2 b) &\not\models \diamond_2 a \wedge \diamond_2 b\end{aligned}$$

- ▶ Inquisitive \vee_3 with alternative-sensitive \diamond_3 gives us **narrow scope** FC inference (Aloni 2007):

$$\begin{aligned}\diamond_3(a \vee_3 b) &\models \diamond_3 a \wedge \diamond_3 b \\ \diamond_3 a \vee_3 \diamond_3 b &\not\models \diamond_3 a \wedge \diamond_3 b\end{aligned}$$

- ▶ But problems under **negation**:

$$\begin{aligned}\neg(\diamond_2 a \vee_2 \diamond_2 b) &\not\models \neg \diamond_2 a \wedge \neg \diamond_2 b \\ \neg \diamond_3(a \vee_3 b) &\not\models \neg \diamond_3 a \wedge \neg \diamond_3 b\end{aligned}$$

Results so far

1. Classical $\vee_1 + \diamond_1$: no FC inference
2. Assertability $\vee_2 + \diamond_2$: only WS epistemic FC with negation problem
3. Inquisitive $\vee_3 + \diamond_3$: only NS FC with negation problem

Desiderata

- ▶ An account of narrow and wide scope FC inferences;
- ▶ For epistemic and deontic modals;
- ▶ Well-behaving under negation.

Three strategies

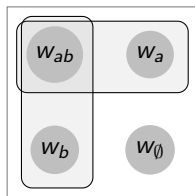
- ▶ **Strategy A**: Extend 1 with a pragmatic account of FC;
- ▶ **Strategy B**: Extend 2 with an account of NS FC;
- ▶ **Strategy C**: Extend 3 with an account of WS FC.

Today focus on strategy B

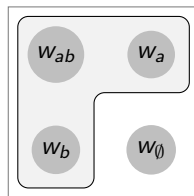
- ▶ **System B**: a semantic account of narrow scope and wide scope FC using enriched version of \vee_2

Strategy C: FC in Inquisitive Semantics

- ▶ Or $\mapsto \vee_3$ (inquisitive)
- ▶ Modals $\mapsto \diamond_3$ (alternative-sensitive)



(g) $a \vee_3 b$



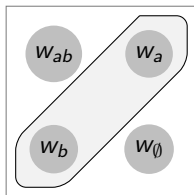
(h) $\neg\neg(a \vee_3 b)$

Relevant results

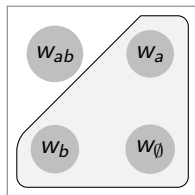
- ▶ Narrow scope FC derived as entailment: $\diamond_3(a \vee_3 b) \models \diamond_3 a \wedge \diamond_3 b$
- ▶ Ways to address the negation problem:
 1. Ambiguity + strongest meaning hypothesis (e.g. Aloni 2007)
 2. Adopt a bilateral system (Roelofsen & Groenendijk, Willer, Fine)
- ▶ But no ready account of wide scope FC:
 - ▶ Epistemic WS FC can be derived by adding semantic structure (Ciardelli et al 2009). But so far no account of deontic WS FC.

Strategy A: FC in state-based pragmatics

- ▶ Or $\mapsto \vee_1$ (classical)
- ▶ Deontic modals $\mapsto \diamond_1$ (relational)
- ▶ Epistemic modals $\mapsto \diamond_2$ (state-based)



(i) $opt(a \vee_1 b)$



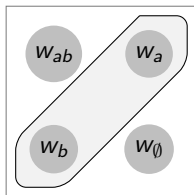
(j) $opt(\diamond_2(a \vee_1 b))$

Implicatures in a state-based semantics

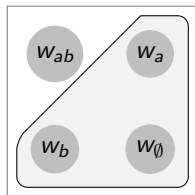
- ▶ Implicatures generated via calculation of **optimal states**
 - ▶ Implicatures of ϕ : what holds in any state in $opt(\phi)$ (Schulz 2005)
(14) $\phi \rightsquigarrow \psi$ iff $\forall s \in opt(\phi) : s \models \psi$ and $\phi \not\models \psi$
 - ▶ Algorithms to compute $opt(\phi)$ based on Gricean principles and/or game theoretical concepts (Aloni 2007, Franke 2009, 2011)
- ▶ **Incorporation** of implicatures in terms of $+I$ operation (Aloni 2012)

Strategy A: FC in state-based pragmatics

- ▶ Or $\mapsto \vee_1$ (classical)
- ▶ Deontic modals $\mapsto \diamond_1$ (relational)
- ▶ Epistemic modals $\mapsto \diamond_2$ (state-based)



(k) $opt(a \vee_1 b)$



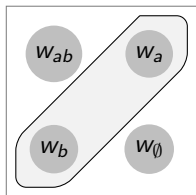
(l) $opt(\diamond_2(a \vee_1 b))$

Relevant results

- ▶ Narrow scope epistemic and deontic free choice derived as implicatures for both \diamond and \square (well behaving under negation);
- ▶ Only deontic FC as embeddable implicatures (Aloni & Franke 2012):
 - ▶ Prediction confirmed by experimental data (Chemla, Geurt et al) and cross-linguistic data on polarity items (Aloni & Port, Fălăuș, Crnič)
- ▶ But no account of WS FC (unless we add covert syntactic structure).

Back to Strategy B

- ▶ $\forall_2 + \diamond_2$ gave us wide scope FC (Hawke & Steiner-Threlkeld 2016), but:
 1. No narrow scope FC;
 2. Problems under negation;
 3. Wide scope FC only derived for epistemic modals (\diamond_2 satisfies epistemic contradiction: $\diamond_2\phi \wedge \neg\phi \models \perp$)
 4. Combination $\forall_2 + \diamond_2$ however not really good for epistemic modals either: $\neg\diamond_2a \vee_2 \neg\diamond_2b$ compatible with $\diamond_2a \wedge \diamond_2b$



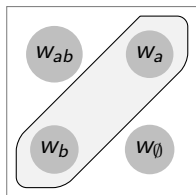
(m) $\models \neg\diamond_2a \vee_2 \neg\diamond_2b$

- ▶ System B attempts to solve all of these problems

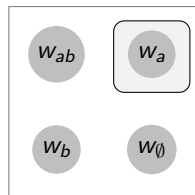
System B: semantic account of wide and narrow scope FC

Disjunction

- ▶ Adopt an enriched version of \vee_2 (non-empty disjunction): $\Rightarrow \vee$
- ▶ A state s supports a **disjunction** $(\phi \vee \psi)$ iff s can be split into two **non-empty** substates, each supporting one of the disjuncts, e.g.



(n) $\models (a \vee b)$



(o) $\not\models (a \vee b)$

- ▶ $[w_a, w_b], [w_{ab}]$ support $(a \vee b)$;
- ▶ but $[w_a]$ no longer supports $(a \vee b)$ [\Leftarrow crucial for **narrow scope FC**]

System B: semantic account of wide and narrow scope FC

Negation facts

- ▶ To account for **negation** facts we adopt a bilateral system:
 - ▶ $s \vdash \phi$ interpreted as “agent in s has enough evidence to assert ϕ ”;
 - ▶ $s \dashv \phi$ interpreted as “agent in s has enough evidence to reject ϕ ”.

Modality

- ▶ A relational (state-based) notion of modality:

$$M, s \vdash \diamond \phi \quad \text{iff} \quad \forall \exists w \in s : M, R^{\rightarrow}(w) \cap \text{info}(\phi) \vdash \phi$$

$$M, s \dashv \diamond \phi \quad \text{iff} \quad \forall \exists w \in s : M, R^{\rightarrow}(w) \dashv \phi$$

- ▶ **Deontic vs epistemic modals:**
 - ▶ Epistemics: R is state-based
 - ▶ Deontics: R is possibly indisputable (e.g. in performative uses)

Outlook of results

- ▶ Narrow scope FC derived because relevant embedded state has to support an enriched disjunction
- ▶ Wide scope FC derived, if R indisputable [state-based \Rightarrow indisput.]
- ▶ Epistemic contradiction derived, if R state-based [epistemics]

System B: definitions

Language

$$\phi := p \mid \neg\phi \mid \phi \wedge \phi \mid \phi \vee \phi \mid \diamond\phi$$

where $p \in A$.

Models

- ▶ $M = \langle W, R, S, V \rangle$, where W is a set of worlds, R is an accessibility relation, S is a set of states (subsets of W), and V is a world-dependent valuation function for A

State-based constraints on accessibility relation

- ▶ R is **indisputable** in M iff $\forall s \in S_M : \forall w, v \in s : R^{\rightarrow}(w) = R^{\rightarrow}(v)$
 \mapsto agents are fully informed about R
- ▶ R is **state-based** in M iff $\forall s \in S_M : \forall w \in s : R^{\rightarrow}(w) = s$
 \mapsto all and only worlds in s are accessible within s

where $R^{\rightarrow}(w) = \{v \mid wRv\}$

System B: definitions

Semantic clauses

$$[M = \langle W, R, S, V \rangle, s, t, t' \subseteq W]$$

$$M, s \vdash p \quad \text{iff} \quad \forall w \in s : V(w, p) = 1$$

$$M, s \nvdash p \quad \text{iff} \quad \forall w \in s : V(w, p) = 0$$

$$M, s \vdash \neg\phi \quad \text{iff} \quad M, s \nvdash \phi$$

$$M, s \nvdash \neg\phi \quad \text{iff} \quad M, s \vdash \phi$$

$$M, s \vdash \phi \wedge \psi \quad \text{iff} \quad M, s \vdash \phi \ \& \ M, s \vdash \psi$$

$$M, s \nvdash \phi \wedge \psi \quad \text{iff} \quad \exists t, t' : t \cup t' = s \ \& \ M, t \nvdash \phi \ \& \ M, t' \nvdash \psi$$

$$M, s \vdash \phi \vee \psi \quad \text{iff} \quad \exists t, t' : t \cup t' = s \ \& \ M, t \vdash \phi \ \& \ M, t' \vdash \psi$$

$$M, s \nvdash \phi \vee \psi \quad \text{iff} \quad M, s \nvdash \phi \ \& \ M, s \nvdash \psi$$

$$M, s \vdash \diamond\phi \quad \text{iff} \quad \forall w \in s : M, R^{\rightarrow}(w) \cap \text{info}(\phi) \vdash \phi$$

$$M, s \nvdash \diamond\phi \quad \text{iff} \quad \forall w \in s : M, R^{\rightarrow}(w) \nvdash \phi$$

where $R^{\rightarrow}(w) = \{v \mid wRv\}$

System B: definitions

Entailment

- ▶ Strong entailment: support-entailment + rejection-entailment

$$\phi \models_S \psi \text{ iff } \forall M, s \in S_M : s \vdash \phi \Rightarrow s \vdash \psi \ \& \ s \dashv \psi \Rightarrow s \dashv \phi$$

- ▶ Weak entailment: support-entailment + dismissal-entailment

$$\phi \models \psi \text{ iff } \forall M, s \in S_M : s \vdash \phi \Rightarrow s \vdash \psi \ \& \ s \dashv \psi \Rightarrow s \not\vdash \phi$$

System B: facts about modals

- ▶ We derive **narrow scope** and **wide scope FC** as (weak) entailments:
 1. $\diamond(a \vee b) \models \diamond a \wedge \diamond b$
 2. $\diamond a \vee \diamond b \models_s \diamond a \wedge \diamond b$ [if R is indisputable]
- ▶ Epistemic vs deontic modals:
 - ▶ Deontic modals: R typically indisputable in performative uses
 - (15) We may either eat the cake or the ice-cream. [+fc, narrow]
 - (16) Either we may eat the cake or the ice-cream. [-fc, wide]
 - (17) You may eat the cake or you may eat the ice-cream.
[+fc, wide] (Fox 2007 & Zimmermann 2000)
 - ▶ Epistemic modals: R is state-based, therefore always indisputable
 - (18) He might either be in London or in Paris. [+fc, narrow]
 - (19) He might be in London or he might be in Paris. [+fc, wide]
 - (20) ?Either he might be in London or in Paris.
 - (21) #It might be raining and it is not raining.
- ▶ We derive **epistemic contradiction**, if R is state-based:
 3. $\diamond a \wedge \neg a \models \perp$ [if R is state-based]

System B: more facts about FC

- ▶ FC effects are **more fine-grained** than in inquisitive semantics:

1. $\diamond(a \vee (a \wedge b)) \models \diamond a \wedge \diamond(a \wedge b)$

2. $\diamond a \vee \diamond(a \wedge b) \models_S \diamond a \wedge \diamond(a \wedge b)$

[if R is indisputable]

- ▶ FC effects also for **plain disjunction** and \Box :

3. $(a \vee b) \models \diamond a \wedge \diamond b$

[if R is state-based]

4. $\Box(a \vee b) \models \diamond a \wedge \diamond b$

($\Box \equiv \neg \diamond \neg$)

- ▶ FC effects disappear under **negation**:

5. $\neg \diamond(a \vee b) \models \neg \diamond a \wedge \neg \diamond b$

6. $\neg(\diamond a \vee \diamond b) \models \neg \diamond a \wedge \neg \diamond b$

7. $\neg(a \vee b) \models_S \neg a \wedge \neg b$

- ▶ But, behaviour under negation is postulated rather than predicted:

- ▶ Allowing to pre-encode what should happen under negation, bilateral systems are more descriptive than explanatory (Cardelli)

System B: some logical properties

- ▶ Double negation law:
 - ▶ $\phi \equiv \neg\neg\phi$
- ▶ De Morgan laws:
 - ▶ $\neg(\phi \vee \psi) \equiv \neg\phi \wedge \neg\psi$
 - ▶ $\neg(\phi \wedge \psi) \equiv \neg\phi \vee \neg\psi$
- ▶ Logic is highly non-standard, e.g. we lose atomic addition:
 - ▶ $a \not\models (a \vee b)$

For comparison

- ▶ Hawke & Steiner-Threlkeld 2016:
 - ▶ $a \models_{HST} (a \vee b)$
 - ▶ $\diamond a \not\models_{HST} (\diamond a \vee \diamond b)$
- ▶ Aloni 2007:
 - ▶ $\phi \models_A (\phi \vee \psi)$
 - ▶ $\diamond a \not\models_A \diamond(a \vee b)$

Summary

- ▶ Three notions of **disjunction** in state-based semantics:
 1. Classical: \vee_1
 2. Team/assertability logic: \vee_2
 3. Inquisitive/truthmaker semantics: \vee_3
- ▶ **System B**: semantic account of FC using an enriched version of \vee_2 :
 - ▶ Narrow scope FC as entailments (well-behaving under negation)
 - ▶ Wide scope FC as entailments (dependent on accessibility relation)
 - ▶ FC effects also for plain disjunction and under \Box
- ▶ Other strategies lacked a ready account of wide scope FC:
 - ▶ **Strategy A**: classical \vee_1 + state-based pragmatics
 - ▶ narrow scope FC as implicatures (both \Diamond and \Box)
 - ▶ only deontic FC as embeddable implicature
 - ▶ no account of wide scope FC (unless we add syntactic structure)
 - ▶ **Strategy C**: inquisitive \vee_3 + alternative-sensitive \Diamond_3
 - ▶ narrow scope FC as entailments
 - ▶ no account of wide scope FC (unless we add semantic structure)

Future work

- ▶ Experimentally test predictions [January 2016, Alexandre Cremers]
- ▶ Logical properties of System B;
- ▶ ...

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